Operators in ultraviolet completions for Electroweak collective symmetry breaking

Nobuhiro Uekusa

5-10 June
Northeastern University
Quadratic divergence

Fermions
Gauge bosons
Gauge-Higgs
Nambu-Goldstone bosons

Chiral Symmetry

Supersymmetry

Gauge

Broken Global

Little Higgs
Quadratic divergence

Fermions

Gauge bosons

Chiral
Symmetry
Gauge

Supersymmetry
Gauge-Higgs

Many Superparticles
KK modes

Nambu-Goldstone bosons
Broken Global
Little Higgs

In this talk
The assumption

**the global symmetry is explicitly broken only when two or more couplings are non-vanishing**

Potential with at most log divergence

Collective symmetry breaking mechanism

Arkani-Hamed, Cohen, Georgi 01, Katz, Nelson, Kaplan, Schmaltz, Cheng, Low, Tucker-Smith, ...
Operator of scalar-gauge interactions

\[ g^2 W_{1\mu} W_{2\mu} h^\dagger h \]

\[ W_{1\mu}, [SU(2)] \text{ gauge bosons} \]

\[ g^2 \left( \frac{W_{1\mu} + W_{2\mu}}{\sqrt{2}} \right)^2 h^\dagger h - g^2 \left( \frac{W_{1\mu} - W_{2\mu}}{\sqrt{2}} \right)^2 h^\dagger h \]
Non-linear $\sigma$ model: self-interaction of the Higgs field

$$|\partial_\mu h|^2 + \frac{1}{f^2} |\partial_\mu h|^2 h^\dagger h$$

$1$-loop

$$\frac{\Lambda^2}{16\pi^2 f^2} |\partial_\mu h|^2$$

Higgs mass

$$\frac{g^2}{2\pi} f \left[ \log \left( \frac{\Lambda}{f} \right) \right]^{1/2} \sim 100 \text{ GeV}$$

$$\Lambda < 4\pi f$$

$$f \sim 1 \text{ TeV}$$

$$\Lambda \sim 10 \text{ TeV}$$
Hadron Physics

QCD

\[ f_\pi \sim 190 \text{ MeV} \]

LOW ENERGY
Nonlinear \( \sigma \) model

HIGH ENERGY
Weakly-coupled gauge theory

Collective breaking

Higgs Physics

\[ f \sim 1 \text{ TeV}(\text{assumption}) \]
At high energies

While the collective symmetry breaking mechanism requires two or more couplings, such a group as $[SU(2)]^2$ could be a subgroup of a single group.

It should be clarified whether operators such as $W_1^\mu W_2^{\mu \dagger} h^2$ can be derived in a gauge theory with a single large group broken to two or more subgroups.

A weakly-coupled renormalizable ultraviolet completion of a little Higgs scenario was proposed by Csaki-Heinonen-Perelstein-Spethmann (08).

Several couplings with the form $W_1^\mu W_2^{\mu \dagger} h^2$ were shown explicitly.

All scalar-gauge couplings to be examined
An operator of no quadratic divergence

Derivation in high energy theory

An extra-dimensional version of symmetry breaking
The single gauge group

\[ \text{SU}(5) \]

\[ \text{SO}(5) \]

Higgs field

fluctuation of the field
developing vev
Higgs sector

SU(5) \rightarrow SO(5)

by vev of a scalar transforming as a symmetric matrix under SU(5)

\[ S = \langle S \rangle + \bar{S} \]

\[ \langle S \rangle = f_S \begin{pmatrix} 1 & 1_2 \\ 1_2 & 1 \end{pmatrix} \]

\[ f_S \equiv \frac{M}{\sqrt{10\lambda_1 + 2\lambda_2}} \]

\[ V_S = -M^2 \text{Tr} [SS^\dagger] + \lambda_1 (\text{Tr} [SS^\dagger])^2 + \lambda_2 \text{Tr}[(SS^\dagger)^2] \]

\[ \bar{S} = iN + R \]

\[ N = \begin{pmatrix} \phi & h & \chi \\ h^T & K_i & h^\dagger \\ \chi^T & h^* \end{pmatrix} \]

\[ R = \begin{pmatrix} \Phi & H & X \\ H^T & K_r & H^\dagger \\ X^T & H^* & \Phi^\dagger \end{pmatrix} \]

Heavy

Integrated out
Potential

\[ V \sim N^4, (N^2 R, N^3 R), \]
\[ (R^2, NR^2, N^2 R^2), (R^3, NR^3), R^4 \]

Effective vertex

No quad div for Higgs field

\[ -V_{\text{Eff}} = \left( \frac{1}{4} g^2 W_1^a W_2^a + \frac{1}{8} g'^2 B_1 B_2 \right) h^\dagger h + \frac{1}{4} g^2 W_1^a W_2^a (\text{Tr} [\phi\phi^\dagger] + \text{Tr} [\chi^2]) \]
\[ + \frac{1}{400} g'^2 (B_1^2 + B_2^2 + 49 B_1 B_2) \text{Tr} \left[ \phi\phi^\dagger \right] - \frac{3}{50} g'^2 (B_1 - B_2)^2 \text{Tr}(\chi^2) \]

Only U(1) part induces quad div
The single gauge group

\[ \text{SU(5)} \]

\[ \text{SO(5)} \]

Scalar fields

5th component of 5D gauge fields

Spatially separated

\[ \left[ \text{SU(2)} \times \text{U(1)} \right]^2 \]

\[ W_{1\mu}^{2}, \quad B_{1\mu}^{2} \]
\[ \delta A_M^a = \partial_M \epsilon^a + g f^{abc} A_M^b \epsilon^c + g f^{\hat{a}\hat{b}\hat{c}} A_M^\hat{b} \epsilon^\hat{c} \]

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_\mu^a )</td>
<td>( N )</td>
</tr>
<tr>
<td>( A_\gamma^a )</td>
<td>( D )</td>
</tr>
<tr>
<td>( A_\mu^a )</td>
<td>( D )</td>
</tr>
<tr>
<td>( A_\gamma^a )</td>
<td>( N )</td>
</tr>
</tbody>
</table>

\( a \) subgroup

\( \hat{a} \) coset
Possible that boundary conditions make the same pattern of symmetry breaking

BUT found that the Higgs field doesn’t have the operator form $g^2 W_{1\mu} W_{2\mu} h^\dagger h$

It has $g^2 W_{1\mu} W_{1\mu} h^\dagger h, \ldots$
Possible that boundary conditions make the same pattern of symmetry breaking

BUT found that the Higgs field doesn’t have the operator form \( g^2 W_{1\mu} W_{2}^{\mu} h^\dagger h \)

It has \( g^2 W_{1\mu} W_{1}^{\mu} h^\dagger h, \ldots \)
[SU(2) X U(1)]^2 gauge bosons couple to Higgs field

Why does the difference appear in the vertex for quad div?

We can say that the same breaking pattern of group is not the same pattern of effective interactions.
vev of scalar fields

\[ \text{SU}(5) \rightarrow \text{SU}(2) \times \text{U}(1) \] w/distinct vev

Global SU(3) \rightarrow \text{Nambu-Goldstone} \rightarrow \text{No potential}

\[ \text{SU}(5) \rightarrow [\text{SU}(2) \times \text{U}(1)]^2 \] Non-zero potential

Two bunches of [SU(2) x U(1)] make potential

at most log div \rightarrow \text{Checked explicitly expected}
boundary conditions

As a breaking at a single boundary

SU(5) $\rightarrow$ SU(2) $\times$ U(1) doesn’t exist

$A^a_\mu$ Dirichlet

$A^a_y$ Neumann

$\delta A^a_y = \partial_y \epsilon^a + g f^{abc} A^b_y \epsilon^c + g f^{\hat{a}\hat{b}\hat{c}} A^{\hat{b}}_y \epsilon^{\hat{c}}$

$\begin{array}{ll}
N & N \\
N & D
\end{array}$

$\partial_y (A^b_y \epsilon^c) \neq 0$

$[SU(2) \times U(1)]^2$ one bunch as a whole

Non-zero potential for one bunch $\rightarrow$ quad div

The pattern SU(5) $\rightarrow$ $[SU(2) \times U(1)]^2$ itself is the same

Accidentally log div? explicit calculation No
Summary

\[ [SU(2) \times U(1)]^2 \]

\[ \xrightarrow{\text{SU(5)}} \]

\[ \xrightarrow{\text{SO(5)}} \]

\[ \xrightarrow{\text{SU(2) \times U(1)}} \]

\[ \text{w/vev} \]

Higgs fields: interactions of the form

\[ g^2 W_{1\mu} W_{2}^{\mu} h^\dagger h \]

\[ \text{Log div} \]

\[ \text{w/boundary conditions} \]

The same breaking pattern

The bunch is different

Quad div