Predictions of Warped Extra Dimensions for Flavor Phenomenology

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1. Motivations for WED:
   - Natural Generation of Hierarchies in Masses and Mixings: Flavor Problem
   - Addressing Gauge-Hierarchy Problem
   - ...

2. Randall-Sundrum Scenario:
   - The Model analyzed
   - New Features in the Flavor Sector (FCNC at Tree Level and non Unitarity of CKM)
   - Neutral Meson Mixing: Theory and Numerics
   - Rare Decays of B and K Mesons: Theory and Numerics

3. Conclusions

Motivations

The SM Flavor Puzzle

\[ Y_D = \mathcal{I}(m_d, m_s, m_b)/v \]
\[ Y_U = V_{CKM}^*(m_u, m_c, m_t)/v \]

\[
Y_D \sim \begin{pmatrix}
10^{-5} & 0 & 0 \\
0 & 0.0005 & 0 \\
0 & 0.026 \\
\end{pmatrix}
\]

\[
Y_U \sim \begin{pmatrix}
10^{-5} & -0.002 & 0.007 + 0.004i \\
10^{-6} & 0.007 & -0.04 + 0.0008i \\
10^{-8} + 10^{-7}i & 0.0003 & 0.96 \\
\end{pmatrix}
\]

Compare to \( g_s \sim 1, \ g \sim 0.6, \ g' \sim 0.3, \ \lambda_{Higgs} \sim 1 \)

SM Yukawa couplings have to exhibit an extremely hierarchical structure, why?
Motivations

Addressing the SM Flavor Problem

- Solution for the equations of motion of the SM fermions in the bulk

\[ f^{(0)}(y, c) = \sqrt{\frac{(1 - 2c) kL}{e^{(1 - 2c)kL} - 1}} e^{\left(\frac{1}{2} (0) ky\right)} \]

Strong dependence on the bulk mass

Zero modes of the Kaluza-Klein towers of quarks (each of them solving the equations of motion in the bulk)

- 4D Yukawas in terms of shape functions:

\[ Y_{ij} \propto \lambda_{ij} f^{(0)}_L(L, c^i) f^{(0)}_R(L, c^j) \]

\[ \lambda_{ij} \text{ assumed to be anarchical and } O(1) \]

5D Yukawas

- Result: slightly different bulk masses of O(1) lead to large hierarchies in \( Y_{ij} \)

Hierarchy of quark masses and mixings explained by a purely geometrical approach

but

Still work in progress: theory for the bulk masses

SM Higgs

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 \]

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Result: slightly different bulk masses of O(1) lead to large hierarchies in \( Y_{ij} \)
Definition of the Model

\[ ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \quad 0 \leq y \leq L \]

\[ e^{-kL} \approx 10^{-16} \]

\[ M_{KK} \approx 2.45ke^{-kL} \approx 2.45 \text{ TeV} \]

Low energy scale

WED with Custodial Protection

T parameter
Csaki et al., 0308038

Agashe et al., 0605341

No protection:
Neubert et al.
0807.4937

SU(2)_L \times U(1)_Y

SU(2)_R \times U(1)_X \times P_{LR}

Zb_L \bar{b}_L coupling

Predictions of WED for Flavor
Definition of the Model

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Low energy scale

- **Gauge bosons:**
  - Gauge eigenstates: \( W^\pm_{L\mu}(++, B_\mu(++) , G^0_{\mu}(++), W^b_{R\mu}(-+), Z_{X\mu}(-+) \)
  - Mass eigenstates: \( W^\pm_{\mu} , W^\pm_{H\mu} , W^\pm_{LR} \)
  - \( A_\mu , A^{(1)}_\mu \)
  - \( Z_\mu , Z_{H\mu} , Z'_{\mu} \)
  - \( G^{(0)}_{\mu} , G^{(1)}_{\mu} \)

  **Gauge bosons of the SM**

- **Higgs boson:**
  - Bi-doublet of \( SU(2)_L \times SU(2)_R \)
  - EWSB mechanism is not specified
  - Resides on the IR brane

- **Fermions:**
  - Different localizations in the bulk of the fermions of the SM:
  - LH down quarks (all three generations) are eigenstates of \( P_{LR} \)
  - RH up quarks (all three generations) are eigenstates of \( P_{LR} \)

**T parameter**

- Csaki et al., 0308038

**WED with Custodial Protection**

- \( Zb_Lb_L \) coupling
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**Predictions of WED for Flavor**
Non Universality & FCNC at Tree Level

- KK tower of heavy gauge bosons...that are all localized towards the IR brane

- Their couplings to SM fermions are **non-universal**...because couplings to SM fermions depend on their localization

\[
\Delta_{L,R} \propto \int_{0}^{L} dy \, e^{k y} \left[ f_{L,R}^{(0)}(y, \psi) \right]^{2} g(y)
\]

**Rotation to mass eigenstates:**

- non universalities \[ \rightarrow \]
- off-diagonal terms

**Flavor Changing Neutral Currents at Tree Level**

- Additional contributions to FCNC at tree level: mixing between SM fermions and KK fermions

**New sources of flavor and CP violation beyond CKM:** **model is non-MFV**
**Theorem:** In theories with $SU(2)_L \times SU(2)_R \times P_{LR}$ gauge symmetry

if a fermion F has $T_L = T_R$, $T^3_L = T^3_R$ or $T^3_L = T^3_R = 0$

then its coupling $ZF \bar{F}$ is SM like

**In RS model:** relation not spoiled by the mixing with KK-fermions

**Consequence for SM fermions:** all the $Z d^i_L \bar{d}^j_L$ and $Z u^i_R \bar{u}^j_R$ couplings are mainly SM like
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Expected small contributions of NP due to the breaking of the $P_{LR}$ symmetry through boundary conditions of the gauge bosons

For the unprotected couplings: action of the **RS-GIM mechanism**
Unitarity of the CKM matrix

\[ V_{CKM}^{RS} = \frac{\sqrt{2}}{g_{4D}} \bar{\tilde{U}}_L \tilde{G}(W) \tilde{D}_L \]
\[ V_{CKM} = \frac{\sqrt{2}}{g_{4D}} \bar{\tilde{U}}_L \tilde{G}(W) \tilde{D}_L \]

Effectively measured CKM matrix

Rotation matrices for Up and Down quarks

\[ \bar{U}_L = \begin{pmatrix} \bar{U}_L \end{pmatrix} \quad \bar{D}_L = \begin{pmatrix} \bar{D}_L \end{pmatrix} \]

Coupling with the charged gauge boson

\[ \tilde{G}(W) = \begin{pmatrix} \tilde{G}(W) \end{pmatrix} \]

\[ \tilde{G}(W)W_\mu^+ \bar{u}_L \gamma^\mu d_L \]

Two sources of non unitarity: due to EWSB we have

Mixing between gauge bosons

- non universality in \( \tilde{G}(W) \)

Mixing between SM and 1\(^{st}\) KK fermions:

- non universality in \( \tilde{G}(W) \)
- \( \bar{U}_L, \bar{D}_L \) not unitary

Numerically: gauge boson contribution predominant

\begin{align*}
V_{CKM} \cdot V_{CKM}^\dagger &= I + O\left( \frac{v^2}{M_{KK}^2} \right) \\
V_{CKM}^\dagger \cdot V_{CKM} &= I + O\left( \frac{v^2}{M_{KK}^2} \right)
\end{align*}

Largest contributions when quarks of the third family involved: 2% effects

Effects below the experimental bounds
Meson Mixing: some Theoretical Aspects

Example: $K^0 - \bar{K}^0$ mixing

**Standard Model**

Process through boxes

![Diagram of Standard Model process](image)

- $Q_1^{VLL}$
- Action of the GIM-mechanism

**Warped Extra Dimensions**

Process already at tree level

- Particles exchanged:
  - KK gluons
  - KK photon
  - $Z, Z', Z''$

![Diagram of WED process](image)

- $Q_1^{VLL}, Q_1^{VRR}, Q_1^{LR}, Q_2^{LR}$

**Operators involved:**

- $Q_1^{VLL} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma_\mu P_L d)$ (also in the SM)
- $Q_1^{VRR} = (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma_\mu P_R d)$
- $Q_1^{LR} = (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma_\mu P_R d)$
- $Q_2^{LR} = (\bar{s}P_L d)(\bar{s}P_R d)$ (only for gluons)
Meson Mixing: some Theoretical Aspects

**Example:** \( K^0 - \bar{K}^0 \) mixing

**Standard Model**

Process through boxes

\[
\begin{align*}
    s & \rightarrow d \\
    d & \rightarrow s
\end{align*}
\]

action of the GIM-mechanism

**Warped Extra Dimensions**

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    Q_1^{VRR} &= (\bar{s}\gamma_\mu P_R d)(\bar{s}\gamma_\mu P_R d) \\
    Q_1^{LR} &= (\bar{s}\gamma_\mu P_L d)(\bar{s}\gamma_\mu P_R d) \\
    Q_2^{LR} &= (\bar{s} P_L d)(\bar{s} P_R d) \quad \text{(only for gluons)}
\end{align*}
\]

In the \( K \) system:

- \( Q_2^{LR} \) dominates gluons
  (Chiral enhancement)

In the \( B \) system:

- Both \( Q_1^{VLL} \) and \( Q_2^{LR} \) are important

EW gauge bosons & gluons

Z boson not relevant
\( \varepsilon_K : \text{Challenging} \) & \( S_{\psi\phi} : \text{NP Room} \)

- Definition of fine tuning taken:

\[
\left( \frac{1}{t} \right)_{BG} = \max_i \frac{d\log(\text{Obs.})}{d\log(x_i)} = \max_i \frac{x_i}{\text{Obs.}} \frac{d\text{Obs.}}{dx_i}
\]


(fitting SM quark masses and CKM elements within 2\( \sigma \))

- Generically \( \varepsilon_K \sim 10^2 \varepsilon_K^{\text{exp}} \)
- Parameter sets with moderate fine tuning and \( \varepsilon_K \sim \varepsilon_K^{\text{exp}} \) exist

\[ M_{KK} \sim 2.45 \text{TeV} \]
Meson mixing: numerics

$\varepsilon_K$: Challenging & $S_\psi\phi$: NP Room

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$M_{KK} \sim 2.45$ TeV

No problem in fitting all the other well measured $\Delta F=2$ observables ($\Delta M_K, \Delta M_d, \Delta M_s, S_{\psi Ks}$) with small fine tuning
Meson mixing: numerics

$\varepsilon_K$: Challenging & $S_{\psi\phi}$: NP Room

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  \]
  \[\text{Barbieri, Giudice, Nucl.Phys.B306:63}\]

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- **Room for new physics**

- **$S_{\psi\phi}$ can be enhanced well beyond the SM prediction $\sim 0.04$**

- **CDF, D$\phi$ hint at $S_{\psi\phi} \sim 0.5$**

- **Strong correlation between the two observables ($S_{\psi\phi}$ and $A_{SL}^s$)**

- **Predictions of WED for Flavor**

No problem in fitting all the other well measured $\Delta F=2$ observables ($\Delta M_K, \Delta M_d, \Delta M_s, S_{\psi Ks}$) with small fine tuning
Example: \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \)

**Standard Model**

First at one loop level

The effective Hamiltonian:

\[ H_{eff}^{SM} \propto V_{ts}^* V_{td} X_{SM} (\bar{s} \gamma_\mu P_L d)(\bar{\nu} \gamma_\mu P_L \nu) \]

Only operator involved
**Example:** \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \)

**Standard Model**

First at one loop level

- The effective Hamiltonian:
  \[
  \mathcal{H}_{\text{eff}}^{\text{SM}} \propto V_{ts}^* V_{td} X_{\text{SM}} \left( \bar{s} \gamma_\mu P_L d \right) \left( \bar{\nu} \gamma_\mu P_L \nu \right)
  \]
  Only operator involved

**Warped Extra Dimensions**

- Additional diagrams at tree level

- Modification of the coefficient of the SM operator
- New operator is induced:
  \[
  \mathcal{H}_{\text{eff}}^{\text{new}} \propto V_{ts}^* V_{td} X^V \left( \bar{s} \gamma_\mu d \right) \left( \bar{\nu} \gamma_\mu P_L \nu \right)
  \]
- Main contributions from the coupling of \( Z \) to **right handed** down quarks

**Predictions of WED for Flavor**

S. Gori
Rare Decays: K physics vs B physics

\[ s \to d \bar{\nu} \nu \quad \text{vs} \quad (b \to d \bar{\nu} \nu \lor b \to s \bar{\nu} \nu) \]

- **Effective Hamiltonian:**

\[
\mathcal{H}_{\text{eff}}^{\text{tot}} \propto V_{tq}^* V_{q_2} \left( X_{SM} + X_{q_1,q_2}^{V-A} \right) (\bar{q}_1 \gamma_\mu (1 - \gamma_5) q_2) (\bar{\nu} \gamma_\mu (1 - \gamma_5) \nu) + \\
+ V_{tq_1}^* V_{q_2} X_{q_1,q_2}^V (\bar{q}_1 \gamma_\mu q_2) (\bar{\nu} \gamma_\mu (1 - \gamma_5) \nu) \]

- Where the new functions:

\[
X_{q_1,q_2}^{V-A,V} \propto \frac{1}{\lambda_t^{(q)}} F_{V-A,V}^{V-A,V} (\Delta_{L}^{\nu \nu}, \Delta_{L,R}^{q_1,q_2}) \]

- **K meson:** \( \lambda_t^{(q)} = V_{ts}^* V_{td} \approx 4 \cdot 10^{-4} \)

- **B mesons:** \( \lambda_t^{(q)} = V_{tb}^* V_{tq} \approx 10^{-2} \), \( q = d, s \)

- **Main Messages:**
  - Non universalities
  - **Expected:** bigger contributions of new physics in the K sector
  - \( X \) function is now complex \( \rightarrow \) new sources of CP violation
Correlations

**B physics vs K physics**

Possible deviations of 15% in the B system; Possible deviations of 200% in the K system.

- **SM prediction**
  \[ Br(B_s \rightarrow \mu^+ \mu^-) = (3.35 \pm 0.32) \cdot 10^{-9} \]

- **Measurement**
  \[ Br(B_s \rightarrow \mu^+ \mu^-) < 4.7 \cdot 10^{-8} \]
**Correlations**

**B physics vs K physics**

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- **Measurement**
  \[ Br \left( B_s \rightarrow \mu^+ \mu^- \right) < 4.7 \times 10^{-8} \]

For the two decays:
Possible deviations of 15% in the B system;
Possible deviations of 200% in the K system

**ΔF=1 vs ΔF=2 observables**

- **SM prediction**
  \[ S_{\psi\phi} \sim 0.04, \quad Br \left( K^+ \rightarrow \pi^+ \nu \bar{\nu} \right) = (8.4 \pm 0.8) \times 10^{-11} \]

- **Measurements**
  \[ S_{\psi\phi} \sim 0.5, \quad Br \left( K^+ \rightarrow \pi^+ \nu \bar{\nu} \right) = (17.3^{+11.5}_{-10.5}) \times 10^{-11} \]

**Difficult to obtain simultaneously large deviations from the SM for both observables**
Conclusions

Warped Extra Dimension with custodial Protection shows:

- **Elegant way to address:**
  - Flavor Problem
  - Gauge-Hierarchy Problem

- **In the Flavor Sector:**
  - Existence of regions of parameter space which:
    - Fit masses of SM quarks and CKM elements
    - Reproduce all the well measured $\Delta F=2$ observables ($\varepsilon_K, \Delta M_K, \Delta M_d, \Delta M_s, S_{\psi K_S}$)
    - Have a **small amount of fine tuning** on the observables → **Address the problem with $\varepsilon_K$**
    - Can predict possible **large deviations** from the SM of observables not measured yet ($S_{\psi \phi}, A_{SL}^s$)

Testability at LHC since $M_{KK} \approx (2 - 3) TeV$ ☺
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- Existence of regions of parameter space which:
  - Fit masses of SM quarks and CKM elements
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  - Can predict possible large deviations from the SM of observables not measured yet ($S_{\psi\varphi}, A^s_{SL}$)

- Restricting to these regions:
  - If future measurements of $S_{\psi\varphi}$ are:
    - large: Branching ratios of $K$ meson decays are small, SM like
    - small: Room for large deviations of $K$ meson decays from SM

  In any case $B$ meson decays can deviate only slightly from the SM

Testability at LHC since $M_{KK} \approx (2 - 3) \, \text{TeV}$

Predictions of the theory