Likelihood analysis of NmSuGra

- Beyond the minimal model
- Likelihood analysis
- Detectability

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Supersymmetry is a robust theory

It explains the origin of

- (inertial) mass: SUSY breaking & radiative dynamics $\rightarrow$ EWSB
- light Higgs boson: $m_h^{\text{tree}} \approx m_Z$ & loop corrections $\rightarrow m_h \lesssim 135$ GeV
- dark matter: conserved $R = (-1)^{3(B-L)+2S} \rightarrow$ LSP is a stable WIMP
- baryonic matter: EW/AD/... baryogenesis $\rightarrow \bar{b}$aryon asymmetry
- gauge unification: sparticle loops $\rightarrow$ unification w/ $M_{\text{GUT}} \sim 10^{16}$ GeV
- naturalness: Higgsinos $\rightarrow$ Higgs mass protected by chiral symmetry
- gravity: gauged supersymmetry $\rightarrow$ supergravity

and more experimental and theoretical puzzles unanswered by the standard particle model
The Minimal Supersymmetric Standard Model (MSSM)

- **Minimal particle content:**
  
  standard fields → superfields

- **Supersymmetry & gauge symmetry →**
  
  all interactions

- **Standard electroweak symmetry breaking →**
  
  particle masses

- **Model parameters are the same as in the standard model**
  
  (with 2 Higgs doublets)

**Superpotential**

\[
W_{\text{MSSM}} = y_u \hat{H}_u \cdot \hat{Q} \hat{U} - y_d \hat{H}_d \cdot \hat{Q} \hat{D} - y_e \hat{H}_d \cdot \hat{L} \hat{E} + \mu \hat{H}_u \cdot \hat{H}_d
\]
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Superpotential

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Supersymmetry → super-partner masses = particle masses
Supersymmetry breaking

However beautiful, attractive and smart SUSY is, she's broken!

Supersymmetry breaking: parametrized by a handful of parameters

Example: minimal supergravity motivated model (mSuGra)

Super-partner masses, with a given spin, are the same at $M_{\text{GUT}}$

- spin 0 spartner masses $\rightarrow M_0$
- spin 1/2 (gaugino) masses $\rightarrow M_{1/2}$
- all trilinear couplings $\rightarrow A_0$
- vacuum expectation values $\rightarrow \tan\beta = \langle H_u \rangle / \langle H_d \rangle$
- electroweak symmetry breaking $\Rightarrow \mu^2 \rightarrow \text{sign}(\mu)$

$$L_{\text{soft}}^{\text{MSSM}} = y_u A_0 H_u \cdot \bar{Q} \tilde{U} - y_d A_0 H_d \cdot \bar{Q} \tilde{D} - y_e A_0 H_d \cdot \bar{L} \tilde{E} + \mu B H_u \cdot H_d + h.c.$$
Problems with the MSSM

µ problem

\[ W_{\text{MSSM}} \supset \mu \hat{H}_u \cdot \hat{H}_d \text{ unnatural} \rightarrow \text{EW size for } \mu \text{ is not justified} \]

Little hierarchy problem

\[ \text{SUSY stabilizes } M_{\text{EW}}, \text{ by protecting } m_h \text{ against } O(M_P) \text{ fluctuations} \]

\[ m_h = \cos^2(2\beta) m_Z^2 + m_{\text{EW}}^2 \left( \log\left( \frac{m_{\text{SUSY}}^2}{m_t^2} \right) + \frac{X_t^2}{m_{\text{SUSY}}^2} \left( 1 - \frac{X_t^2}{12 m_{\text{SUSY}}^2} \right) \right) \]

\[ \text{protection is effective if } m_{\text{SUSY}} \sim M_{\text{EW}} \]

\[ \text{but EW precision data } \rightarrow m_{\text{SUSY}} \sim O(1 \text{ TeV}) \]

Electroweak fine-tuning problem

\[ \max_i\left( \frac{1}{m_Z} \frac{dm_Z}{dp_i} \right) \text{ large in most constrained MSSM scenarios} \]

Dark matter fine-tuning problem

\[ \max_i\left( \frac{1}{\Omega} \frac{d\Omega}{dp_i} \right) \text{ large in most constrained MSSM scenarios} \]
Singlet extensions of the MSSM

The root of the $\mu$, hierarchy & fine-tuning problems is the Higgs sector

- extending the EWSB sector of the MSSM, these problems are alleviated

- in the (n,N,S)MSSM the $W \supset \mu \hat{H}_u \cdot \hat{H}_d$ dynamically generated by

\[ W \supset \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d \]

- all these fields ($H_i$ and $S$) acquire vev.s at the weak scale

- little hierarchy and fine-tunings are alleviated

Next-to-minimal MSSM: $\mathcal{W}_{\text{NMSSM}} = \mathcal{W}_{\text{MSSM}, Y} + \lambda \hat{S} \hat{H}_1 \cdot \hat{H}_2 + \frac{\kappa}{3} \hat{S}^3$

combined with mSuGra $\rightarrow$ universality: fixes all NMSSM parameters, but $\lambda$

5 free parameters (and a sign):

\[ M_0, M_{1/2}, A_0, \tan\beta, \lambda, \text{sign}(\mu) \]

A single parameter extension of mSuGra solving several MSSM problems
NmSuGra

Discreet symmetries of super- & Kahler potentials: $Z_3 \times Z_2^{MP}$

solve domain wall problem

Next-to-minimal MSSM: $W_{NMSSM} = W_{MSSM} + \lambda \hat{S} \cdot \hat{H}_1 \cdot \hat{H}_2 + \frac{\kappa}{3} \hat{S}^3$

New parameters: $\langle S \rangle, \lambda, \kappa, A_\lambda, A_\kappa, m_S$

SUSY breaking: $m_{SuGra} \rightarrow$ universality: fixes $A_\kappa = A_\lambda = A_0$

9 parameters left: $M_0, M_{1/2}, A_0, \langle H_1 \rangle, \langle H_2 \rangle, \langle S \rangle, \lambda, \kappa, m_S$

3 minimization eq. & $v^2 = \langle H_1 \rangle^2 + \langle H_2 \rangle^2$ eliminates 4 para &

$tan\beta = \langle H_1 \rangle / \langle H_2 \rangle$, $\mu = \lambda \langle S \rangle$ exchanges $\beta$ and $\mu$ with 2 para →

5 free parameters (and a sign):

$M_0, M_{1/2}, A_0, tan\beta, \lambda, sign(\mu)$

A single parameter extension of mSuGra – no new dimensionful para.s
Dark matter in NmSuGra

\[ \tilde{\chi}_1 = N_{11} \tilde{B} + N_{12} \tilde{W}_3 + N_{13} \tilde{H}_u + N_{14} \tilde{H}_d + N_{15} \tilde{\Phi} \]

1: focus point, 2: Higgs resonances, 3: $\tilde{\chi}_1$ co-annihilation, 4: $\tilde{\chi}_1$ co-ann.

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Bayesian inference

We are interested in the marginalized posterior probability that, given a set of data, the parameters of a theory acquire certain values

\[ P(p_i|D;T) = \int P(P|D;T) \prod_{j \neq i} dp_j \quad i=1...N_{parameters} \]

\( P(P|D;T) = \text{posterior distribution} \leftarrow \text{Bayes' theorem} \)

\[ P(P|D;T) = \frac{P(D|P;T) \Pi(P|T)}{E(D|T)} \]

\( L(D|P;T) = \text{likelihood, at which the theory explains the data} \)

\[ L(D|P;T) = \prod_i \exp(-\chi_i^2/2)/\sqrt{2\pi \sigma_i} \]

\[ \chi_i^2 = (d_i - t_i(p_i))^2/(\sigma_{i,\text{exp}}^2 + \sigma_{i,\text{the}}^2) \quad i=1...N_{data\ points} \]

\( \Pi(P|T) = \text{prior, describes the a priori distribution of } P \text{ in } T \)

\( E(D|T) = \text{evidence, only normalizing factor} \leftarrow \text{for now} \)
Experimental input

Experimental data, constraining supersymmetry, available today

- LEP: lower limits on spartner, Higgs masses & cross sections
  - most constraining: $m_{\tilde{W}_1} > 103.5$ GeV & $m_h > 114.4$ GeV
- Tevatron: as for LEP & upper limit on $\text{Br}(B_s \to l^+ l^-)$
- $b$ factories: $\text{Br}(b \to s \gamma), \text{Br}(B^+ \to l^+ \nu_l), \Delta M_d, \Delta M_s, ...$
- $g_\mu-2$: anomalous magnetic moment of muon
  - plays strong role: constraining high $M_0$ and $M_{1/2}$
- WMAP: WIMP abundance upper limit
  - very important: excluding significant para-space
- CDMS: WIMP-proton elastic recoil
- GLAST: WIMP self annihilation in the galaxy and beyond
Typical likelihood distributions - low $\tan \beta$

no $g_{\mu-2}$ constraint imposed
Typical likelihood distributions - mid $\tan \beta$

no $g_{\mu-2}$ constraint imposed
Typical likelihood distributions - high $\tan \beta$

no $g_{\mu-2}$ constraint imposed
Profile likelihood distributions

likelihood is suppressed by $g_{\mu-2}$ at high $M_0$ ...
Profile likelihood distributions

... and at high $M_{1/2}$
Posterior probabilities

enhanced at high $M_0$ ...
Posterior probabilities

... and toward high $M_{1/2}$
Posterior probability maps

\[ P(p_i | D; T) \sim \int L(P | D; T) \prod_{j \neq i} dp_j \quad i = 1 \ldots N \text{ parameters} \]
$P(p_i|D;T)$ is enhanced by Higgs resonances and focus point at high $\tan\beta$
LHC reach for NmSuGra

Posterior probability

\[ M_{1/2} \text{ (TeV)} \]

\[ M_0 \text{ (TeV)} \]

--- 68 % CL    --- 95 % CL    --- LHC 100 fb^{-1} reach

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Direct dark matter detection in NmSuGra

Marginalized posterior probability

$\log_{10}(\sigma_{SI}/\text{pb})$

$m_{\tilde{\chi}_{1}^0}$ (GeV)

--- 68% CL

--- 95% CL

--- CDMS 25 kg reach
Summary

- Observations suggest the existence of a wider theory containing the standard model of particle physics
- Supersymmetry is a robust candidate for this theory
- Present experiments prefer moderate values of the dimensionful parameters in the simplest supersymmetric models
- The minimal and next-to-minimal models can be discovered at the CERN Large Hadron Collider with a good chance
- The LHC and near future underground dark matter searches are guaranteed to discover even next-to-minimal models!
- There's a beautiful complementarity between the LHC and direct dark matter detection experiments
The Fat Lady Sang