SUSY Parameter Determination at LHC using Kinematic Fits

- University of Hamburg -

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Motivation

After discovery of SUSY → Parameter determination:

- **Often used technique:** Combination of sensitive observables
  - Dilepton mass edge, cross sections, branching ratios ... → Fitting of model parameters
- **Alternative:** Reconstruction of full kinematics of SUSY events → access to masses
  - For one event: More unknowns (LSP momenta, SUSY masses) than constraints ($p_T$ balance, invariant masses)
  - For more events: some unknowns (SUSY masses) are common → Problem can be over constrained
- **Possible approach:** Number of unknowns equals number of constraints → Look at parameter space covered by solutions (B. McElrath et. al.)
- **Our approach:** **Constrained least square fit of many events taking into account uncertainties of measurements**

Outline: ① Definition of over constrained problem
  ② Alternative fitting technique (genetic algorithm)
  ③ First results
  ④ Further discriminating variables
Signal Cascade & Potential Problems

Our hope: significant fraction of SUSY events with rather similar decay chains (degenerated masses, dominant branching ratios)

Potential problems: • Many jets → huge combinatorial bg (7 jets: 1260 combinations)
  • Effect of SM and SUSY backgrounds
  • Detector resolution and acceptance
  • Initial and final state radiation
  • No perfect mass degeneration
  • Width of virtual particles

For $N$ events:
$N \times 21$ local measurements (7 jets)
$4 \times$ global unknowns (SUSY masses)
$N \times 6$ local unknowns (2 LSPs)
$N \times 7$ local constraints:
  • $p_x, p_y$
  • $1 \times M_{\text{gluino}}, 2 \times M_{\text{squark}}, 2 \times M_{\text{chargino/neutralino}}$

Over constrained for $N > 4$
Non-linear Constrained Fits

Method for constrained fits: **Method of Lagrangian Multiplier**

Invariant mass constraints in general not linear

→ Linearization via Taylor expansion

→ Iterative solution

**Problems:**

- Linearization of constraints only good approximation “near” solution → if “away” from solution iterative procedure might results in **too large** or **too small steps**, or even **wrong direction**

- Definition of convergence criterion

**Used fitting code: KinFitter**

- C++ implementation ... (V. Klose and J. Sundermann)
- ... of **ABC FIT** from ALEPH collaboration (O. Buchmüller and J. B. Hansen)
- Additional modifications (step scaling and scaling of constraints)
Alternative Approach: Genetic Algorithm

- Formulation of constraints as additional $\chi^2$ term $\rightarrow$ “cost function”
- Interpret cost function as $\chi^2 \rightarrow$ full error propagation:

$$\chi^2_M = \left( \frac{M_{\text{inv}}(j_1, j_2, j_3) - M}{\sigma} \right)^2$$

with

$$\sigma^2 = \sum_{i=1}^{N_j} \left( \frac{\partial M_{\text{inv}}}{\partial j_i} \right)^2 \cdot \sigma_{j_i}^2 + \Gamma^2_M$$

Minimize cost function: gradient, simplex, LBFGS, simulated annealing and genetic algorithm (GA)

**GA:** Final state 4-momenta are genome of individual; jet combination is one additional gene. Fitness function (here $\chi^2$) defines which individual is fittest

**Algorithm:**
1) Create first generation of individuals (starting population)
2) Select best fitting individuals
3) Create new individuals by selecting randomly two parents and inherit randomly genes from either one or other parent
4) For each child mutate each genome with small probability
5) Back to step 2) until convergence

**Advantage:** no linearization needed $\Leftrightarrow$ **Disadvantage:** high computational cost
Proof of Principle: Semi-Leptonic $\bar{t}t$

Counting unknowns and constraints:

- 4 jets + 1 lepton = 15 measured parameters
- 1 neutrino = 3 unmeasured parameters
- 4 constraints ($p_x, p_y, 2 \times M_W$)

Combinatorics:

- No b-tagging used
- → 12 possible jet configurations

Event generation and detector simulation:

- **Pythia6** generated events including ISR and FSR
- Each final state jet smeared according to typical jet momentum and angular resolutions at ATLAS/CMS
- Jet/lepton selection cuts: Four jets and one lepton with
  - $p_T > 20$ GeV
  - $|\eta| < 3.0$
Resolution of fitted neutrinos:

- **Genetic algorithm:**
  - Right jet combinations has smallest $\chi^2$ for 301 of 860 events

- **KinFitter:**
  - Converged for 667 of 860 events
  - Right jet combinations has smallest $\chi^2$ for 231 events

**Scenario:**
- No bg from other processes
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance

Similar performance of both methods for neutrino resolution
Proof of Principle: Semi-Leptonic $t\bar{t}$ (cont.)

$M_{\text{had}}^\text{top}$

Comparable and reasonable results for both fitting techniques:
- Small differences of probability distributions:
  - Different treatment of constraints

Small deviations from flat distributions:
- Combinatorics
- Bias due to jet cuts (no perfect momentum balance)
SUSY Sample

- **mSUGRA test point:**
  - **Parameters:** \( m_0 = 230 \text{ GeV}, m_{1/2} = 360 \text{ GeV} \)
    \[ A_0 = 0, \tan \beta = 10, \text{sign} \mu = + \]
  - **Masses:** \( m_{\tilde{q}} \approx 810 \text{ GeV}, m_{\tilde{g}} \approx 860 \text{ GeV} \)
    \[ m_{\chi^\pm_1} \approx 273 \text{ GeV}, m_{\chi^0_1} \approx 147 \text{ GeV} \]
  - **Cross section at LHC:** \( \sigma_{\text{tot}} = 7.8 \text{ pb(LO)} \)
  - **Branching ratios:**
    \[ Br(\chi^0_2 \rightarrow h^0 \chi^0_1) \approx 85\% \]
    \[ Br(\chi^{\pm}_1 \rightarrow W^{\pm} \chi^0_1) \approx 97\% \]

- **Pythia6** generated events including ISR and FSR

- Each final state jet smeared according to typical jet momentum and angular resolutions at ATLAS/CMS

- Jet selection cuts: 7 jets with
  - \( p_T > 20 \text{ GeV} \)
  - \( |\eta| < 3.0 \)

→ Dominant background of other SUSY processes (S/B ~ 1/30)
Fit of SUSY Events with Genetic Algorithm

- No SM bg
- No SUSY bg
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance
- Mass hypothesis = true masses

**First step:** Reconstruct SUSY events with true mass hypothesis

- Reasonable resolution of unmeasured particles
- Neutralino Starting momenta: set to fulfill chargino mass constraint
- Cross check: Fulfilled constraints after fit
Reduction Combinatorial Background

- 25% of best hypothesis have right jet combinatorics (started with 1/1260 ~ 0.08%)
- Typical wrong combinations are exchanged branches

Event number

right combinatorics

good $\leftrightarrow \chi^2 \rightarrow$ bad
Similar probability distribution of SUSY background:

- “Signal like” cascade topologies, e.g. decays via heavier charginos or neutralinos
- Signal cascades but different squark mass (3rd generation)
- Signal cascades but one soft jet replaced by ISR jet
- Huge jet combinatorics

Fit probability distribution flat over large range

Highest probabilities missing:
- Convergence criteria?
- Underestimation of constraints' uncertainties?

- No SM bg
- Full SUSY bg
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance
- Mass hypothesis = true masses
• Fix gluino and neutralino mass to true values
• Vary two remaining masses (squark and chargino)
• Average <best fit probability for each event>

- Concordance between averaged fit probability and true values
- Reason for deviation: larger chargino mass leads to more kinematic freedom for neutralino
- Significantly smaller average fit probability for background

- No SM bg
- Full SUSY bg
- Full combinatorial bg
- ISR and FSR
- Detector resolution and acceptance
- Scan mass hypothesis
Adding signal and background events:

For typical SUSY scenario: kinematic fits using momentum and invariant mass constraints → completely dominated by the SUSY background (S/B ~ 1/30)

→ Difficult to determine masses (might be more promising if cascade is more frequent)

→ Further discriminating variables / constraints needed
Angular Distributions

- Huge combinatorial background → Large invariant mass combinations, e.g.

  - In rest frame of SUSY particles: angular distribution $\cos \theta^*$ of decay products with respect to flight direction of decaying particle should be ~isotropic (for spin 0)
  
  - $\cos \theta^*$ for typical background 4-vector configurations are not uniformly distributed (smaller angles preferred)

Many decay angles in SUSY cascades

→ Use event kinematics to reduce combinatorial bg reduction
Summary:

- Genetic algorithm yields comparable results to Lagrangian Multipliers and is well suited for highly non-linear problems.
- Kinematic fits provide a powerful tool to reconstruct SUSY cascades.
- Invariant mass constraints reduce combinatorial background of signal cascades (0.08% → ~25%).
- Combinatorial SUSY background dominant for studied mSUGRA scenario → further discriminating variables needed, e.g. $\cos \theta^*$.

Outlook:

- Include final states with leptons (reduced combinatorics).
- Add combined likelihood of kinematic variables to fitness function (further reduction of combinatorial SUSY background).
- Fit more than one hypothesis.
- Study other models than one specific SUSY scenario.
Backup
Method of Lagrangian Multipliers

The first step is to formulate each constraint as equation that equals zero: \( c(\vec{a}) = 0 \)

Definition of a new function \( L \) for \( P \) constraints

\[
L(\vec{y}, \vec{a}) = \chi^2 + 2 \cdot \sum_{i=1}^{P} \lambda_i c_i(\vec{a})
\]

with Lagrangian Multipliers \( \lambda_i \)

Partial derivative w.r.t. \( \lambda_i \):

\[
\frac{\partial L}{\partial \lambda_i} = 2 \cdot c_i(\vec{a}) = 0
\]

→ Solution must fulfill the constraints!

Partial derivatives w.r.t. \( a_i \):

\[
\frac{\partial L}{\partial a_i} = \frac{\partial \chi^2}{\partial a_i} + 2 \cdot \sum_{j=1}^{P} \lambda_i \cdot \frac{\partial c_j}{\partial a_i} = 0 \rightarrow \nabla \chi^2 = -2 \cdot \sum_{j=1}^{P} \lambda_j \nabla c_j
\]

→ Gradients of \( \chi^2 \) function \( \nabla \chi^2 \) and of constraint function \( \nabla c \) must be parallel!

→ \( \chi^2 \) function has a local minimum on the constraint contour!

\( \chi^2 \) of solution should follow a \( \chi^2 \) distribution with \( N - P \) degrees of freedom (\( N \): number of measured parameters)
Genetic Algorithm (cont.)

Mutation typ A: exchange of whole branches

Mutation typ B: exchange of two jets
In typical Susy scenarios: \( m_{\chi_{1}^{\pm}} - m_{\chi_{1}^{0}} \gtrsim m_{W} \)

→ Small relative momentum of \( W \) and \( \chi^{0} \)

Assume same direction of \( W \) and \( \chi^{0} \) and adjust \( \chi^{0} \) momentum to fulfill mass constraint

\[
0 \overset{!}{=} f(x) = m_{\chi_{1}^{\pm}}^{2} - \left( E_{W} + \sqrt{m_{\chi_{0}}^{2} + (x \cdot p_{W})^{2}} \right)^{2} - (1 + x)^{2} p_{W}^{2}
\]

In general there are zero, one or two solutions for \( x \)

- 0: Take \( x \) with smallest \( |f(x)| \) (set derivative \( df/dx \) to 0)
- 1: Ok ... but in practice this never happens
- 2: Choose solution with smaller \( |x| \)

Method can be generalized for any invariant mass constraint with one unmeasured particle and an arbitrary number of measured particles
Evolution of Fitness Function

One Single Event:

Overcome local minimum
Shapes look different for signal and background