Anomaly Mediated Supersymmetry Breaking Demystified

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Spontaneous Supersymmetry Breaking

- Hamiltonian

\[ H = P^0 = \frac{1}{4} \left( Q_1 Q_1^\dagger + Q_2 Q_2^\dagger + Q_1^\dagger Q_1 + Q_2^\dagger Q_2 \right) . \]

- SUSY breaking means that vacuum \( |0\rangle \) is not invariant under SUSY transformations,

\[ Q_\alpha |0\rangle \neq 0 \quad \text{and} \quad Q_\alpha^\dagger |0\rangle \neq 0. \]

- So the vacuum energy is \textbf{positive} in the case of Spontaneous SUSY breaking.

\[ \langle 0|H|0 \rangle > 0. \]
Visible sector SUSY breaking is not viable. For instance,

\[ m_{\tilde{e}_1}^2 + m_{\tilde{e}_2}^2 = 2m_e^2, \]

is not acceptable.

SUSY is broken in the hidden sector, and its effect is mediated to the visible sector.

→ Mediation mechanism is required.
Hidden Sector SUSY breaking

- Planck-scale-mediated SUSY breaking (PMSB).

\[ m_{\text{soft}} \sim \frac{\langle F \rangle}{M_P}, \quad \sqrt{\langle F \rangle} \sim 10^{10} - 10^{11} \text{ GeV}. \]

\[ \rightarrow m_{\text{soft}} \sim \frac{\Lambda^3}{M_P^2}, \text{ for gaugino condensation.} \quad \Lambda \sim 10^{13} \text{ GeV}. \]
Gauge-mediated SUSY breaking (GMSB).

- Messengers: New chiral supermultiplets those couple to SUSY breaking VEV \( \langle F \rangle \) and also have gauge interaction.

- Soft SUSY breaking terms are derived from the loop diagrams involving some messenger particle.

\[
m_{soft} \sim \frac{\alpha_a}{4\pi} \frac{\langle F \rangle}{M_{mess}}
\]

- Flavor blind (No FCNC) and \( \sqrt{\langle F \rangle} \sim 10^4 \) GeV.
SUSY breaking source is only non-zero $\langle F \rangle$ (Superconformal compensator).

Assume that no dimensionful parameters in the action. → SUSY breaking is not transmitted to the visible sector.

But in the anomaly loop, SUSY breaking is manifest.
Conformal Anomaly - Massless QCD

- For scale transformations $x \rightarrow e^\rho x$,

$$ q_j(x) \rightarrow e^{3\rho/2} q_j(e^\rho x), $$

$$ A^a_m(x) \rightarrow e^\rho A^a_m(e^\rho x). $$

- The current is $J^m_{\text{scale}}$,

$$ J^m_{\text{scale}} = x_n T^{mn}, \quad \partial_m J^m_{\text{scale}} = T^n_n = 0, $$

- By quantum effect, non-zero trace is generated,

$$ \tilde{S}_{\text{eff}} = \int d^4x \mathcal{O} T^m_m, \quad T^m_m = \frac{\beta_{\text{QCD}}(g)}{2g} F^a_{nl} F^{anl}, $$
In conformal supergravity, the gravitational superfield is

\[ \mathcal{H}^m(x, \theta, \bar{\theta}) = \theta \sigma^a \bar{\theta} e^m_a(x) + \frac{i}{2} \bar{\theta} \bar{\theta} \psi^m(x) - \frac{i}{2} \theta \theta \bar{\psi}^m(x) + \frac{1}{4} \theta \theta \bar{\theta} \hat{\nu}^m(x). \]

Here \( \hat{\nu}^m(x) \), \( e^m_a(x) \), and \( \psi^m_\alpha(x) \) are a \( U(1)_R \) gauge transformations, vierbein and gravitino, respectively, with DOF 3, 5, and 8.

This multiplet couples to supercurrents, - energy momentum tensor \( T_{mn} \), supersymmetry current \( S^m_\alpha \) (along with its conjugate, \( \bar{S}^m_\dot{\alpha} \)) and \( R \)-current \( j^R_m \).
Suppose that they are anomalous,

$$\xi_\alpha \equiv \gamma_m S^m_\alpha \neq 0, \quad \hat{t} \equiv T^m_m \neq 0, \quad \hat{r} \equiv \partial^m j^R_m \neq 0.$$ 

They form chiral anomaly supermultiplet with auxiliary fields $a$ and $b$,

$$\mathcal{X}(x, \theta) \equiv A(x) + \sqrt{2} \theta \xi(x) + \theta \theta \mathcal{F}(x), \quad \bar{D} \mathcal{X} = 0,$$

where $A = a + ib$ and $\mathcal{F} = \hat{t} + i \hat{r}$.
Chiral Anomaly Supermultiplet and Chiral compensator

- Superfield which couples to CASM?
  
  → the trace anomaly \( \hat{\tau} \) - dilaton, \( \varrho(x) = \frac{1}{2} \ln \det[e^m_a] \).
  
  → the \( U(1)_R \) anomaly \( \hat{r} \) - the local \( R \)-symmetry, \( \delta(x) \) (NGB of \( U(1)_R \)).
  
  → the supersymmetry anomaly \( \xi_\alpha \) - the dilatino
  
  \( (\sim \bar{\Psi}_\alpha(x) = \sigma^m_{\dot{a}} \bar{\psi}_{\dot{a}}^m(x)) \).

- So, the chiral compensator,

\[
\chi^3(x, \theta) \equiv e^{2\varrho(x)+2i\delta(x)}[1 + \sqrt{2}\theta \bar{\Psi}(x) + \theta \theta M^*(x)],
\]
Chiral Anomaly Supermultiplet and Chiral compensator

- Chiral compensator is invariant measure,

\[ d^4x' d^2\theta' \chi'^3(x', \theta') = d^4xd^2\theta \chi^3(x, \theta). \]

- We can comprehend this property by decomposing the chiral compensator as

\[ d^4xd^2\theta \chi^3(x, \theta) = \left\{ d^4xe^{2\varrho(x)} \right\} \left\{ d^2\theta e^{2i\delta(x)} [1 + \sqrt{2}\theta^\alpha \bar{\Psi}_\alpha(x) + \theta\theta M^*(x)] \right\}. \]

- Invariant measure + trace anomaly \(\rightarrow\) Action,

\[ S_\chi = \int d^4x d^2\theta \chi^3(x, \theta) \chi(x, \theta) + c.c. \]
Chiral Anomaly Supermultiplet and Chiral compensator

- Effective theory viewpoints,

\[ \phi(x, \theta) = M_{\text{pl}} \chi^3(x, \theta) = M_{\text{pl}} e^{2q + 2i\delta} \left[1 + \sqrt{2} \theta^\alpha \tilde{\Psi}_\alpha + \theta \theta M^* \right]. \]

\[ S_{\chi} = \int d^4x \, d^2\theta \frac{1}{M_{\text{pl}}} \phi(x, \theta) \chi(x, \theta) + c.c. \]

- Non-vanishing VEV \( \langle \phi \rangle / M_{\text{pl}} = 1 + \theta \theta \langle M \rangle \) (with \( \langle q \rangle = \langle \delta \rangle = 0 \)) leads to soft terms in the visible sector.

- Component fields,

\[ S_{\chi} = \int d^4x \left[ e^{2q + 2i\delta} (M^* A + \tilde{\Psi} \xi + F) + c.c. \right]. \]

\( \rightarrow \) First term with nonzero vev \( M \) leads soft term.

\( \rightarrow \) \( \langle \delta \rangle - \hat{F} \hat{F} \sim \) axion
Assume that

\[ \partial^m j^R_m = 0, \]
\[ \xi_\alpha = \gamma_m S^m_\alpha \neq 0, \quad \dot{t} = T^m_m \neq 0. \]

- \( j^R_m, \xi_\alpha, \) and \( \dot{t} \) form Linear Anomaly supermultiplet (LASM).

Most generally, LASM is written

\[
L(x, \theta, \bar{\theta}) = C(x) + i \theta \bar{\Xi} - i \bar{\theta} \Xi + \theta \sigma^m \bar{\theta} j^R_m(x) \\
- \frac{1}{2} \theta \theta \bar{\theta} \bar{\sigma}^m \partial_m \Xi(x) - \frac{1}{2} \bar{\theta} \bar{\theta} \theta \sigma^m \partial_m \bar{\Xi}(x) - \frac{1}{4} \theta \theta \bar{\theta} \bar{\theta} \Box C(x).
\]

- \( \dot{t} \sim \Box C, \quad \xi_\alpha \sim \sigma^m_{\alpha \dot{\alpha}} \partial_m \bar{\Xi}_{\dot{\alpha}}. \)
Couples what? - Using the following supersymmetric generalization of $U(1)_{R}$ gauge transformations:

$$\mathcal{V} \rightarrow \mathcal{V} + \Lambda + \Lambda^{+},$$

with $\Lambda$ being a chiral field ($\bar{D}\Lambda = 0$).

The most generally,

$$\mathcal{V}(x, \theta, \bar{\theta}) = s(x) + i\theta \omega(x) - i\bar{\theta} \bar{\omega}(x)$$

$$+ \frac{i}{2} \theta \theta [p(x) + iq(x)] - \frac{i}{2} \bar{\theta} \bar{\theta} [p(x) - iq(x)]$$

$$- \theta \sigma^{m} \bar{\theta} \nu_{m}(x) + i\theta \theta [\bar{\tau}(x) + \frac{i}{2} \bar{\sigma}^{m} \partial_{m} \omega(x)]$$

$$- i\bar{\theta} \bar{\theta} [\tau(x) + \frac{i}{2} \sigma^{m} \partial_{m} \bar{\omega}(x)] + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} [\mathbf{d}(x) + \frac{1}{2} \Box s(x)].$$
In the Wess-Zumino gauge where $s = \omega = p = q = 0$, and we identify $\nu_m$, $\tau_\alpha$ and $d$ are the gauge field, gaugino, and $D$-term, respectively, for $U(1)_R$.

$U(1)_R$ gauge transformations

$$\nu_m \rightarrow \nu_m - i \partial_m (|\Lambda| - |\Lambda^+|).$$

Note that $d$ and $\tau_\alpha$ are gauge invariant component fields in $\mathcal{V}$.

$\sim$ dilaton and dilatino?
Roughly, we can identify

\[ \nu_{\alpha\dot{\alpha}} = \psi_\alpha \bar{\psi}_{\dot{\alpha}}, \]
\[ \tau_\alpha = \bar{\psi}_\alpha M, \]
\[ d = M^* M, \]

where \( \nu_{\alpha\dot{\alpha}} = \nu_m \sigma^m_{\alpha\dot{\alpha}} \).

The action is

\[ S_L = \int d^4x d^2\theta d^2\bar{\theta} \mathcal{V} L. \]

Remind that \( \mathcal{V} = 0 \) and \( L = 2 \).
Linear Anomaly Supermultiplet and Vector Superfield

- This transforms as

\[ \int d^4x d^4\theta \mathcal{V} L \rightarrow \int d^4x d^4\theta (\mathcal{V} + \Lambda + \Lambda^+) L, \]

\[ = \int d^4x d^4\theta \mathcal{V} L + \int d^4x d^2\theta \Lambda (\bar{D}^2 L) + \int d^4x d^2\bar{\theta} \Lambda^+ (D^2 L). \]

- The condition should be satisfied

\[ D^2 L = \bar{D}^2 L = 0, \]

\[ L \text{ is anything but the } R\text{-current superfield}. \]
Linear Anomaly Supermultiplet and Vector Superfield

- Component fields,

\[ S_L = \int d^4x \left( \nu^m j^R_m + \frac{1}{2} \mathbf{d} C + \tau \Xi + \bar{\tau} \bar{\Xi} \right). \]

→ The second term is only relevant for the soft sfermion masses. (footprints of conformal anomaly)
The MSSM

- Set $\langle M^* \rangle = m_{3/2}$ and $d = m_{3/2}^2$.

- The action,

$$S = \int d^4x \, d^2\theta d^2\bar{\theta} \, \phi_i^+ e^{2gV} \phi_i$$

$$+ \left[ \int d^4x \, d^2\theta \left( \frac{1}{4} W^{a\alpha} W_{a\alpha} + \frac{1}{3!} y^{ijk} \phi_i \phi_j \phi_k \right) + h.c. \right].$$
The MSSM

- CASM from the gauge kinetic term, with the supersymmetrization of massless QCD,

\[
\mathcal{X} = \frac{\beta(g)}{2g} W^{a\alpha} W^{a\alpha},
\]

\[
\beta(g) = -\frac{g^3}{16\pi^2} [3C_2(G) - FC_2(\mathcal{R})],
\]

where \(C_2(G) = N\) and \(C_2(\mathcal{R}) = 1/2\) for \(SU(N)\). Then

\[
M_\lambda = \frac{\beta(g)}{g} m_{3/2}.
\]
The MSSM

- CASM form Yukawa interaction.

\[ S_{\text{eff}} = \int d^4x \ d^2\theta \ \frac{1}{3!} \ y^{ijk} \left\{ \frac{1}{32\pi^2} \left[ y^{*irs} y^{i'rs} - 4g^2 C_2(\mathcal{R}) \delta_{ii'} \right] \frac{\mu^{-2\epsilon}}{2\epsilon} \phi_i \phi_j \phi_k + (\text{cyclic}) \right\} \]

- Replacing \( \mu \) by \( e^{-\varrho} \mu \) and then picking up the linear term in \( \varrho \),

\[ S_{\text{eff}} = -\int d^4x \ d^2\theta \ \varrho \ \frac{1}{3!} \ y^{ijk} \left\{ \frac{1}{32\pi^2} \left[ y^{*irs} y^{i'rs} - 4g^2 C_2(\mathcal{R}) \delta_{ii'} \right] \phi_i \phi_j \phi_k + (\text{cyclic}) \right\} \]

\[ = \int d^4x \ d^2\theta \ \varrho \ \frac{1}{3!} \left( \gamma_i + \gamma_j + \gamma_k \right) y^{ijk} \phi_i \phi_j \phi_k. \]

- The CASM reads

\[ \mathcal{X} = \frac{1}{3!} \left( \gamma_i + \gamma_j + \gamma_k \right) y^{ijk} \phi_i \phi_j \phi_k. \]
The MSSM

- A term is given by
  \[ A_{ijk} = -(\gamma_i + \gamma_j + \gamma_k) y^{ijk} m_{3/2}. \]

- We supersymmetrize the dilaton. That is, the dilaton is transformed into the chiral compensator:
  \[ e^{2\phi} \rightarrow \chi^3 = e^{2\phi + 2i\delta} [1 + \sqrt{2}\theta\bar{\Psi} + \theta\theta M^*]. \]

  Under this transformation,
  \[
  \ln \mu^2 \rightarrow \ln \mu^2 + \ln \chi^{-3} \\
  = \ln \mu^2 - 2\phi - 2i\delta - \sqrt{2}\theta\bar{\Psi} - \theta\theta M^*. 
  \]

- We can derive the interaction dilaton(ino)(axion) and CASM like this.
The MSSM

- LASM, evaluated at two loop,

\[ L = -\frac{1}{4} \dot{\gamma}_i \phi_i^+ \phi_i, \]
\[ \dot{\gamma}_i = \frac{\partial \gamma_i}{\partial \ln \mu}. \]

Then,

\[ m_i^2 = \frac{1}{4} \dot{\gamma}_i m_3^{2/3}. \]
Summary and Outlook

- We present the novel field-theoretical understanding of the Anomaly Mediated Supersymmetry breaking.
- It is more understandable compared with the conventional spurion method.
- We can reproduce the results with the anomaly superfields interactions.
- In addition to that, we can write down the dilaton-interaction action, which can be used for study of hidden sector.
- More study on the hidden sector and Einstein supergravity will be pursued.