General Aspects of Gauge Mediation

SUSY 09

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Based on:
M. B., P. Meade, N. Seiberg, and D. Shih
and
M. B. and Z. Komargodski, to appear

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Motivation for (low scale) SUSY breaking

- Hierarchy problem
- Gauge coupling unification
- Assuming R-parity, have a DM candidate
Complications

- Led to a hidden sector paradigm
- Communication of SUSY breaking
- Explicit ‘soft’ breaking in SSM
Problems for generic soft terms

• Lots of terms

• SUSY flavor problem

• SUSY CP problem

• $\mu$ problem

• All these problems are related to the mediation of SUSY breaking
Enter gauge mediation

• Distinctive phenomenology

• Calculable examples

• Natural solution to SUSY flavor problem

• Not much to say about SUSY CP problem

• Naively, still have $\mu$ problem; work done to address this:
  Z. Komargodski and N. Seiberg; C. Csaki, A. Falkowski, Y. Nomura, and T. Volansky; G. F. Giudice, H. D. Kim, and R. Rattazzi;...
Summary of what is to follow...

- Review of minimal gauge mediation (MGM)
- Review and reformulation of general gauge mediation (GGM)
- IR and UV properties of the GGM correlation functions
- Existence of messenger models of GGM; questions of calculability
- Spontaneously broken symmetries and massless particles in GGM
- Further analysis of the IR
MGM

M. Dine and W. Fischler; M. Dine, A. Nelson, Y. Nir, Y. Shirman,...

\[ W = \lambda X \phi \bar{\phi} \] (1)

- \( X = \langle X \rangle = M + \theta^2 F \) is a SUSY-breaking spurion
- Messengers \( \phi, \bar{\phi} \) transform in a vector-like rep of \( G_{SM} \)
- Bosonic messengers get SUSY breaking masses \( |M|^2 \pm F \) through the coupling to \( X \)
MGM soft spectrum

- Gaugino masses at 1-loop

\[ M_{\lambda r} \sim N_r \cdot \frac{\alpha_r}{4\pi M} \frac{F}{M} + \mathcal{O}\left(\frac{F^3}{M^5}\right) \]  

(2)

- Sfermion masses at 2-loops

\[ M^2_{\tilde{f}} \sim \sum_{r=1}^{3} c_r(\tilde{f}) \cdot N_r \cdot \left(\frac{\alpha_r}{4\pi}\right)^2 \left(\frac{F}{M}\right)^2 + \mathcal{O}\left(\frac{F^4}{M^6}\right) \]  

(3)
MGM phenomenology

• Gravitino LSP

• Colored sparticles more massive

• For $N_f \sim 1$ gaugino and sfermion masses ‘similar’

• Bino or slepton NLSP

• Unification of gauge couplings $\Rightarrow$ gaugino mass unification

• **Question:** Are these general predictions of GM?
Introduction to GGM
P. Meade, D. Shih, and N. Seiberg
M. B., P. Meade, D. Shih, N. Seiberg

- Theory comprised of a hidden sector ($\mathcal{H}$) and a visible sector ($\mathcal{V}$); sectors decouple in the limit $\alpha_{SSM} \rightarrow 0$

- $\mathcal{H}$ spontaneously breaks SUSY at a scale $M$

- $\mathcal{H}$ has a weakly gauged global symmetry $\mathcal{G} \supset \mathcal{G}_{SSM}$

- Data of $\mathcal{H}$ visible to SSM summarized in current supermultiplet $\mathcal{J}$.

- Idea: work exactly in contributions of $\mathcal{H}$ and perturbatively in $\alpha_{SSM}$.

- In principle, we encompass strongly coupled hidden sectors
The $\mathcal{J}$ current multiplet

- For simplicity we will assume $\mathcal{G} = U(1)$ in what follows.

- Lowest component, $J = J^\dagger$, satisfies

\[
\{ Q_\alpha, [Q_\beta, J] \} = 0 \tag{4}
\]

- We further define

\[
\begin{align*}
  j_\alpha & \equiv -i [Q_\alpha, J] \\
  \bar{j}_\dot{\alpha} & \equiv i [\bar{Q}_{\dot{\alpha}}, J] \\
  j_\mu & \equiv -\frac{1}{4} \sigma^\alpha_\mu (\{ \bar{Q}_{\dot{\alpha}}, [Q_\alpha, J] \} - \{ Q_\alpha, [\bar{Q}_{\dot{\alpha}}, J] \}) \\
  \end{align*}
\tag{5}
\]

- SUSY algebra and (4) $\Rightarrow \partial_\mu j^\mu = 0$.

- Include messengers in $\mathcal{H}$ with contribution $J = \sum_i q_i |\phi_i|^2$.
\[ \langle J J \rangle \text{ 2-point functions} \]

- Assume that goldstino is the only massless particle in \( \mathcal{H} \); everything else scale \( M \).

- In this case, Lorentz invariance and current conservation \( \Rightarrow \) non-zero 2-point functions have the form

\[
\begin{align*}
\langle J(p)J(-p) \rangle &= C_0 (p^2 / M^2) \\
\langle j_\alpha(p)\bar{j}_\alpha(-p) \rangle &= -\sigma^\mu_\alpha p_\mu C_{1/2} (p^2 / M^2) \\
\langle j_\mu(p)j_\nu(-p) \rangle &= -(p^2 \eta_{\mu\nu} - p_\mu p_\nu) C_1 (p^2 / M^2) \\
\langle j_\alpha(p)j_\beta(-p) \rangle &= \epsilon_{\alpha\beta} M B_{1/2} (p^2 / M^2)
\end{align*}
\]  

(6)

- The \( C_\alpha \)'s are real and \( B \) is complex. Note that \( B = 0 \) unless R-symmetry broken.

- When SUSY is preserved, \( C_0 = C_{1/2} = C_1 \) and \( B = 0 \).
Gauging the symmetry

- Couple the current to a vector superfield; in WZ gauge, we have

\[ \mathcal{L}_{int} = 2g \int d^4 \theta \mathcal{J} \mathcal{V} + ... = g(JD - \lambda j - \bar{\lambda} j - j^\mu V_\mu) + ... \]  \hspace{1cm} (7)

- Integrate out \( \mathcal{H} \)

\[ \delta \mathcal{L}_{eff} = \frac{g^2}{2} \tilde{C}_0(0) D^2 - g^2 \tilde{C}_{1/2}(0) i \lambda \sigma^\mu \partial_\mu \bar{\lambda} - \frac{g^2}{4} \tilde{C}_1(0) F^2 \]
\[ - \frac{g^2}{2} (M \tilde{B}_{1/2}(0) \lambda^2 + c.c.) + ... \]  \hspace{1cm} (8)

- Leading coefficients are interpreted as change in beta function due to integrating out hidden sector matter
GGM soft masses

\[ \delta \mathcal{L}_{\text{eff}} = \frac{g^2}{2} \tilde{C}_0(0) D^2 - g^2 \tilde{C}_{1/2}(0) i \lambda \sigma^\mu \partial_\mu \bar{\lambda} - \frac{g^2}{4} \tilde{C}_1(0) F^2 \]

\[ -\frac{g^2}{2} (M \tilde{B}_{1/2}(0) \lambda^2 + \text{c.c.}) + ... \]  

\[ (9) \]

• Integrating out \( H \) generates a Majorana gaugino mass term (assume R-symmetry explicitly broken in \( H \))

\[ M_\lambda = g^2 M B_{1/2}(0) \]  

\[ (10) \]

• We also generate masses for the scalars

\[ M^2_{\tilde{f}} = g^4 c(\tilde{f}) A \]  

\[ (11) \]
with $A$ given by the following linear combination of current 2-point functions

$$A = -\int \frac{d^4 p}{(2\pi)^4} \frac{1}{p^2} \left( 3\tilde{C}_1(p^2/M^2) - 4\tilde{C}_{1/2}(p^2/M^2) + \tilde{C}_0(p^2/M^2) \right)$$

(12)

- **Puzzle:** Why does $A$ converge? In principle, could have dangerous contributions from UV and IR.

- **IR infinities:** Assumed all particles but goldstino massive, so only possible source is goldstino. Can kill goldstino poles with a holomorphic messenger parity, $\mathcal{J} \rightarrow -\mathcal{J}$. This also kills potentially dangerous contributions from the 1-point function, $\langle \mathcal{J} \rangle$. **We will also have more to say about the IR later.**
UV finiteness of GGM soft masses
M. B., P. Meade, N. Seiberg, D. Shih

- Using our definition of the current multiplet, we can show that
  \[ M_\lambda = -\frac{1}{4} g^2 \int d^4 x \langle Q^2 (J(x) J(0)) \rangle \]  \hspace{1cm} (13)

- Can also show that
  \[ \langle |Q|^4 J(x) J(0) \rangle = -8 \partial^2 \left( x^{-4} (C_0(x) - 4C_{1/2}(x) + 3C_1(x)) \right) \]  \hspace{1cm} (14)

- At the UV fixed point, the \( JJ \) OPE gives
  \[ J(x) J(0) = \frac{c_1}{x^4} 1 + \frac{c_O}{x^{4-\Delta}} O + \ldots \]  \hspace{1cm} (15)
Therefore, in the absence of massless particles mixing with $\mathcal{J}$, the OPE implies that there are no boundary contributions in the sfermion mass integral, the sfermion masses are finite, and

$$M_{\tilde{f}}^2 = -\frac{1}{128\pi^4} g^4 c(\tilde{f}) \int d^4x \log(x^2 M^2) \langle \bar{Q}^2 Q^2 (J(x)J(0)) \rangle$$  \hspace{1cm} (16)

- Comments: $M$ drops out, i.e.

$$\int d^4x \log M^2 \langle |Q|^4 J(x)J(0) \rangle = 0$$  \hspace{1cm} (17)

- There are two dual interpretations of this statement:

- First, in the IR, an observer in a massive theory does not need a regulator to compute observables.
• Second, in the UV, an observer in a theory that flows to a superconformal fixed point doesn’t need a regulator either. In particular, there is no UV ‘supertrace.’

• Additional comments: our formulas make clear that the particle masses vanish in a SUSY vacuum.

• Generalization of small SUSY breaking notion of gaugino mass as an F-term and sfermion mass as a D-term.

• Let’s now turn to some more concrete particle physics by generalizing our above discussion to the MSSM gauge group and finding phenomenological implications...
MSSM gauge group and mass relations

- Now have three complex $B_r$ and three real $A_r$; again, assume $\mathcal{J} \rightarrow -\mathcal{J}$ symmetry

\[
M_{\chi_r} = g_r^2 MB_r, \quad m_f^2 = \sum_{r=1}^{3} g_r^4 c_r(\bar{f}) A_r
\]  

(18)

with $f = Q, U, D, L, E$

- Hence, there must be two mass relations

\[
\text{Tr} Y m^2 = m_Q^2 - 2m_U^2 + m_D^2 - m_L^2 + m_E^2 = 0
\]

\[
\text{Tr}(B - L) m^2 = 2m_Q^2 - m_U^2 - m_D^2 - 2m_L^2 + m_E^2 = 0
\]  

(19)

- Mass relations hold at $\mathcal{O}(\alpha^2)$ and at scale $M$
What else do we know about the masses?

• Not much! Don’t even know the sign of the sfermion mass-squareds in general!
E. Poppitz and S. Trivedi; Y. Nakayama, M. Taki, T. Watari, and T. T. Yanagida

• **Question:** are the mass relations the only constraints in GGM?

• Can we explore the parameter space with weakly coupled theories? Do we need strongly coupled theories?

• **Claim:** we can explore the full parameter space just with weakly coupled messenger models!
• GGM consists of a 9 real dimensional parameter space. 3 complex $B_r$’s $\Rightarrow$ CP problem

• We’ll assume CP invariance of the hidden sector and only discuss the 6 real dimensional GGM parameter space
**Messenger theory generalities**

- Messenger models are not UV complete and so come with a cutoff, $\Lambda$, where new physics enters.

- To ensure calculability of soft masses, want to only consider renormalizable messenger sectors.

- Particularly obvious problems if $\text{STr} \mathcal{M}^2 \neq 0$. Then have a log divergence coming from UV

\[
\int \frac{d^4 p}{p^2} (3\tilde{C}_1 + ...) \sim \int \frac{d^4 p}{p^4} \text{STr} \mathcal{M}^2 + ... \quad (20)
\]
Covering the GGM parameter space: messenger generalities

- \( \phi^i, \bar{\phi}^i \) transform in \( \mathbb{R} \oplus \bar{\mathbb{R}} \) rep of \( G \).

\[
V_{\text{mass-terms}} = (\bar{\psi}^T M_F \psi + \text{c.c.}) + \Phi^\dagger M_B^2 \Phi \tag{21}
\]

with \( \Phi = (\phi \bar{\phi}^*)^T \)

\[
M_B^2 = \begin{pmatrix}
M_F^\dagger M_F + \xi & F \\
F^\dagger & M_F M_F^\dagger + \bar{\xi}
\end{pmatrix}
\]

- \( \xi \) represents diagonal type breaking
• \( \xi = \xi^*, \quad \tilde{\xi} = \tilde{\xi}^*, \quad F = F^* \)

• Invariance under \( \phi^i \leftrightarrow \tilde{\phi}^i \) and \( V_Y \rightarrow -V_Y \) \( \Rightarrow \) \( \xi = \tilde{\xi} \) and \( F = F^T \)

• \( \text{STr} \mathcal{M}^2 = 0 \Rightarrow \text{Tr} \xi = 0 \)

• Assume messengers come in reps of \( SU(5) \)

• \( R = \bigoplus_R (n_R \times R) \Rightarrow \) conditions hold for each irrep
Covering the GGM parameter space
M.B., P. Meade, N. Seiberg, and D. Shih

- Would like to cover the full space $A_r, B_r \in \mathbb{R}^+$. 

- Extend results of L. Carpenter, M. Dine, G. Festuccia, and Mason

- We’ll consider case that messengers come in $SU(5)$ multiplets; restrict attention to the following reps

\[
\begin{align*}
\bar{5} &= D \oplus L, & 10 &= Q \oplus U \oplus E
\end{align*}
\]

- We can cover the parameter space if we have enough different SSM reps and allow for diagonal type SUSY breaking.
• It turns out that the two simplest solutions are $2 \times (5 \oplus \bar{5}) \oplus 10 \oplus \bar{10}$ and $2 \times (10 \oplus \bar{10})$

• Therefore sum rules really are all there is!

• Have gauge coupling unification without gaugino mass unification

• Exotic NLSP...
Massless particles in GGM
M. B. and Z. Komargodski, to appear

- Hidden sector QFTs can have several massless particles! We’ve already come across the goldstino in this talk.

- In $U(1)'$ extensions of the MSSM there is also a massless pion.

- In R-symmetric models of gauge mediation, there is a massless partner of the gaugino. This leads to a Dirac mass—as opposed to the Majorana mass discussed above.

- In theories with a spontaneously broken R-symmetry, there is an R-axion.
• How do these affect observables? How do we get a general handle on the physics? Some work in this direction has been initiated in e.g., K. Intriligator and M. Sudano; K. Benakli and M. Goodsell, ...
Taxonomy of massless particles

• The massless particles are generally the particles associated to broken symmetries: the Goldstone(s) and the goldstino(s)

• And the unbroken symmetries: fermions needed to saturate anomalies.
Effects of massless particles

• The most important effect for us is that the sfermion masses may in principle be divergent. I.e. a symptom of

\[
\int d^4x \log M^2 \langle |Q|^4 J(x)J(0) \rangle \neq 0 \tag{23}
\]

which receives corrections from the massless states.

• Therefore, we need to understand how and when to re-sum the contributions from the relevant correlators.

• **Idea:** To get a handle on the physics, study correlation functions of the broken symmetry generators with the current that generates the symmetry \( G \) that the sfermions are charged under.
Broken symmetry current correlators

- Let me just give a flavor of the interesting physics:

- SUSY is broken, so we would like to study the contributions of the goldstino to the sfermion mass physics. To do this, we examine

\[
\langle S_{\alpha}^\mu(x) j_\beta(0) \rangle = \tilde{c} \epsilon_{\alpha\beta} \partial^\mu \left( \frac{1}{x^2} \right) + \sigma_{\alpha\beta}^{\mu\nu} \partial_\nu \tilde{g}(x^2 M^2) \quad (24)
\]

- Using our definition of \( J \), we can show that \( \tilde{c} = 0 \). Then, flowing to the IR, where \( S_{\alpha}^\mu \sim \sigma_{\alpha\beta}^\mu \tilde{G}^\beta \), we can show that the goldstino does not mix with \( j_\beta \). Hence, it cannot contribute to the bad IR behavior of the sfermion mass integrand. This argument also shows that the goldstino can never be the fermionic partner of the Goldstone boson of a spontaneously broken non-R symmetry.
When the R-symmetry is spontaneously broken, it is also crucial to determine the contributions of the R-axion to the current two-point functions. For example, we should examine

$$\langle j_\mu^R(x) J(0) \rangle = c_R \partial_\mu (x^{-2}) \quad (25)$$

Using the R-invariance of $J$ we can show that $c_R = 0$. Flowing to the IR, where $j_\mu^R \sim \partial_\mu R$, we can then show that the R-axion doesn’t couple to $J$.

$G$ itself might be Higgsed. We know the corresponding pion then couples to the spin-1 correlator $\langle j_\mu(x) j_\nu(0) \rangle$. But can it couple to any other of the correlators contributing to the sfermion masses?

Only other possibility is that it couples to $\langle J(x) J(0) \rangle$. Then, we should study

$$\langle j_\mu(x) J(0) \rangle = c \partial_\mu (x^{-2}) \quad (26)$$
where we’ve used current conservation and Lorentz invariance.

- Using the $G$-invariance of $J$ one can show that the above correlator vanishes. Then, flowing to the IR we can show that the pion doesn’t couple to $J$. 
The upshot

• In the end it is possible to get a general handle on the IR behavior of the sfermion mass integrand in the presence of massless particles that couple to the relevant correlators that feed into the scalar masses.

• We obtain general mass formulae that look like GGM formulae but that generally have threshold corrections from the vector multiplet masses induced by resumming IR divergences due to the various relevant massless states. There are also RG/supertrace corrections due to running from the vector multiplet scale(s) to the hidden sector scale.

• In R-symmetric cases, there is novel physics because we can have fermions that saturate R-anomalies and then generate Dirac
masses for the gaugino upon weakly gauging $G$ in addition to the above-discussed Majorana masses

- In the higgsed $U(1)'$ case, the sum rules can be modified; some discussion in M. Luo et. al.
Conclusions

• Can make precise and general statements about the nature of the soft observables in gauge mediation.

• Gauge mediation is described by sum rules—a smoking gun to look for at LHC.

• It is worth taking the corresponding parameter space seriously since we can cover it with very simple models.

• Various extensions of the GGM framework can lead to different sets of sum rules.

• Many interesting future directions!