SUSY-QCD CORRECTIONS TO DARK MATTER ANNIHILATIONS

Karol Kovařík

Laboratoire de Physique Subatomique et de Cosmologie
Grenoble, France

in collaboration with B. Herrmann and M. Klasen


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1. Motivation
2. Annihilation to quarks
3. SUSY-QCD radiative corrections
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Motivation for Dark Matter

- **First observational hints**
  - Velocity dispersion and rotation curves
    
    
    [Zwicky 1933, Rubin et al. 1970]

- **CMB anisotropies**
  - Cosmological parameters from WMAP mission
    
    
    [Komatsu et al. (WMAP) 2008]

    \[ \Omega_{\text{tot}} = 1.005 \pm 0.034 \]
    \[ \Omega_{\text{CDM}} = 0.223 \pm 0.013 \]

- **Structure formation**
  - Cold dark matter needed to explain large structures
    
    
    [Blumenthal et al. 1984]
Motivation for precision calculations

Ultimate goal - consistency test between Cosmology & Beyond the SM physics

→ experimental cosmology gets more precise COBE-WMAP-Planck
→ before the start of LHC cosmology provides constraints

COBE 1989

WMAP 2002
Ultimate goal - consistency test between Cosmology & Beyond the SM physics

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Before the start of LHC cosmology provides constraints COBE 1989 WMAP 2002

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![COBE 1989](image1)
![WMAP 2002](image2)
![Planck 20??](image3)

**Planck satellite will deliver new cosmological data in near future**
- more precise theoretical predictions needed to match experimental improvements
Motivation for precision calculations

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**Planck satellite will deliver new cosmological data in near future**

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**Higher order corrections can have important contributions to cross sections**

→ QCD corrections significant due to strong coupling constant
→ modification of preferred regions in parameter space
Dark Matter & SUSY

Dark matter relic density required to agree with WMAP+SN+BAO data

\[ 0.1097 \leq \Omega_{\text{CDM}} h^2 \leq 0.1165 \]

[Hinshaw et al. 2008]
Dark Matter & SUSY

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Number density of relic particle governed by the Boltzmann equation

\[
\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{ann}} v \rangle (n^2 - n_{\text{eq}}^2) \quad \Omega_{\text{CDM}} h^2 \propto n \propto \frac{1}{\langle \sigma_{\text{ann}} v \rangle}
\]

Thermal average involves velocity distribution of the relic particle

\[
\langle \sigma_{\text{ann}} v \rangle = \int dv \tilde{f}(v)\sigma_{\text{ann}} v = \int ds f(s)\sigma_{\text{ann}}(s)
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Cross section \( \sigma_{\text{ann}} \) includes all annihilation and co-annihilation processes
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Public codes perform a calculation of the relic density for given scenario

\[ \rightarrow \text{DarkSUSY (only neutralino)} \quad \rightarrow \text{micrOMEGAs (all kinds of LSP)} \]

[Gondolo et al. 2004] [Bélanger et al. 2006]
Annihilation into quarks

- Cross section includes s-channel Z-boson & Higgs boson, t & u-channel squark exchange

\[ \sigma \nu = a + bv^2 + \mathcal{O}(v^4) \]

- Annihilation into light quarks (all except top) *always kinematically allowed*
  → dominant for light neutralino

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- Non-relativistic limit of annihilation cross-section

  \[ \sigma v = a + b v^2 + \mathcal{O}(v^4) \]

- Leading coefficient in annihilation to quarks proportional to the mass of the quark
  → light quarks of 1st & 2nd generation suppressed
  → top quark final states dominant if allowed
Annihilation into quarks

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Different scenarios lead to different dominant contributions
→ mSUGRA - Higgs exchange dominates & \( \tan \beta \) important parameter
→ no gaugino unification - Z-boson or squark exchange dominate
Annihilation into quarks in mSUGRA

Contribution of quark final states in mSUGRA $m_0-m_{1/2}$ planes

- $\tan\beta=10, A_0=-1500$ GeV, $\mu>0$
- $\tan\beta=50, A_0=0$, $\mu>0$

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Annihilation into quarks in mSUGRA

**Contribution of quark final states in mSUGRA m$_0$-m$_{1/2}$ planes**

- **tanβ=10, A$_0$=-1500 GeV, μ>0**
  - Crossed areas indicate no EWSB
  - Levels of 80%, 60%, 40%, 30%, and 10% are shown
  - $b\rightarrow s\gamma$ transitions are marked

- **tanβ=50, A$_0$=0, μ>0**
  - Crossed areas indicate no EWSB
  - Levels of 80% and 60% are shown

**Interesting regions:**
- Focus point region (tt dominated)
- Bulk & A-funnel region (bb dominated)

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Annihilation into quarks beyond mSUGRA

- Relax \textit{GAUGINO MASS UNIFICATION} - compatible with gauge coupling unification

- Parameters $M_1, M_2, M_3$ independent at GUT scale

\[ x_1 = \frac{M_1}{M_2}, \quad x_3 = \frac{M_3}{M_2} \]
Annihilation into quarks beyond mSUGRA

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x_1 = \frac{M_1}{M_2} \quad \quad x_3 = \frac{M_3}{M_2}
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- Gluino parameter $M_3$ very influential
  - decrease in $M_3 \rightarrow$ decrease in $M_{Hu}$ and squark masses
  - low $M_{Hu} \rightarrow$ low $\mu \rightarrow$ larger higgsino fraction of $\tilde{\chi}_1^0$
Annihilation into quarks beyond mSUGRA

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  - decrease in $M_3 \rightarrow$ decrease in $M_{H_u}$ and squark masses
  - low $M_{H_u} \rightarrow$ low $\mu \rightarrow$ larger higgsino fraction of $\tilde{\chi}_1^0$

- First scenario - small squark masses & low $\tan\beta \rightarrow$ squark exchange

  [S.Martin 2007]
Annihilation into quarks beyond mSUGRA

- Relax **Gaugino Mass Unification** - compatible with gauge coupling unification

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  \[ x_1 = \frac{M_1}{M_2}, \quad x_3 = \frac{M_3}{M_2} \]

- Gluino parameter $M_3$ very influential
  - decrease in $M_3 \rightarrow$ decrease in $M_{\text{Hu}}$ and **squark masses**
  - low $M_{\text{Hu}} \rightarrow$ low $\mu \rightarrow$ larger higgsino fraction of $\tilde{\chi}_1^0$

- **First scenario** - small squark masses $\&$ low $\tan \beta \rightarrow$ squark exchange
  
  [S. Martin 2007]

- **Second scenario** - large higgsino component of $\tilde{\chi}_1^0 \rightarrow Z$ exchange
  
  [Bertin, Nezri, Orloff 2002]
SUSY-QCD corrections

**Virtual loop corrections:** On-shell renormalization

**Real emission corrections:** Dipole subtraction method

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Virtual loop corrections:
- loops are calculated in $\overline{\text{DR}}$ regularization scheme
- UV divergence removed by On-shell renormalization

Real corrections:
- $\overline{\text{DR}}$ regularization scheme for IR divergence in loops & gluon radiation - poles
- Dipole subtraction method combines virtual & real part - cancel IR divergence

\[
\sigma_{1\text{-loop}} = \left[ \sigma_V + \int d\sigma_{\text{aux}} \right]_{\varepsilon=0} + \int \left[ d\sigma_R - d\sigma_{\text{aux}} \right]_{\varepsilon=0}
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\]

Higgs exchange & Yukawa couplings

→ Higgs resonances lead to enhanced annihilation cross-section & correct relic density
→ Higgs boson decays to fermions well known
→ QCD corrections up to $O(\alpha_s^4)$ included
→ SUSY-QCD corrections to bottom Yukawa coupling known to be important for large $\tan\beta$
Numerical results - mSUGRA

- mSUGRA parameter values:
  \( m_0 = 1500 \text{ GeV}, \quad m_{1/2} = 130 \text{ GeV} \)
  \( \tan \beta = 10, \quad A_0 = -1500 \text{ GeV}, \quad \text{sgn} \, \mu = + \)

- Relic density:
  \( \Omega h^2 = 0.116, \quad \bar{\text{bb}} = 86\% \)
Numerical results - mSUGRA

- **mSUGRA parameter values:**
  
  \[ \begin{align*}
  m_0 &= 1500 \text{ GeV}, \\
  m_{1/2} &= 130 \text{ GeV} \\
  \tan \beta &= 10, \\
  A_0 &= -1500 \text{ GeV}, \\
  \text{sgn} \mu &= +
  \end{align*} \]

- **Relic density:**
  
  \[ \Omega h^2 = 0.116, \quad \bar{b}b = 86\% \]
Numerical results - mSUGRA

- mSUGRA parameter values:
  \[ m_0 = 5300 \text{ GeV}, \ m_{1/2} = 625 \text{ GeV} \]
  \[ \tan \beta = 10, \ A_0 = -1500 \text{ GeV}, \ \text{sgn} \ \mu = + \]

- Relic density:
  \[ \Omega h^2 = 0.110, \ \bar{\tau} t = 72\% \]
Numerical results - mSUGRA

mSUGRA parameter values:
- $m_0 = 5300$ GeV, $m_{1/2} = 625$ GeV
- $\tan\beta = 10$, $A_0 = -1500$ GeV, $\text{sgn} \mu = +$

Relic density:
- $\Omega h^2 = 0.110$, $\tilde{t} \tilde{t} = 72\%$
Parameter values:

- $m_0 = 320 \text{ GeV}$, $M_2 = 700 \text{ GeV}$, $x_1 = 2/3$, $x_3 = 1/3$
- $\tan\beta = 10$, $A_0 = -350$, $\text{sgn}\,\mu = +$

Relic density:

- $\Omega h^2 = 0.114$, $\bar{t}t = 79\%$
Numerical results - beyond mSUGRA

- **Parameter values:**
  \[ m_0 = 320 \text{ GeV}, \quad M_2 = 700 \text{ GeV}, \quad x_1 = 2/3, \quad x_3 = 1/3 \]
  \[ \tan \beta = 10, \quad A_0 = -350, \quad \text{sgn} \mu = + \]

- **Relic density:**
  \[ \Omega h^2 = 0.114, \quad \bar{\tau} \tau = 79\% \]
Parameter values:

- $m_0 = 1500$ GeV, $M_2 = 600$ GeV, $x_1 = 1$, $x_3 = 4/9$
- $\tan\beta = 10$, $A_0 = 0$, $\text{sgn} \, \mu = +$

Relic density:

- $\Omega h^2 = 0.104$, $\bar{\tau} \tau = 50\%$
Numerical results - beyond mSUGRA

**Parameter values:**

\[ m_0 = 1500 \text{ GeV}, \ M_2 = 600 \text{ GeV}, \ x_1 = 1, \ x_3 = 4/9 \]
\[ \tan\beta = 10, \ A_0 = 0, \ \text{sgn} \mu = + \]

**Relic density:**

\[ \Omega h^2 = 0.104, \ \bar{\tau}t = 50\% \]
Conclusions

Higher-order corrections relevant for constraining SUSY parameter space using dark matter data

SUSY-QCD corrections to neutralino annihilation to bottom & top quarks ~ 20-30%

For analysis with precision of a few %
- include full EW corrections to ALL processes
- study Spectrum code differences & improve