How fine-tuned is a large Muon EDM from Flavor?

G. Hiller, K. Huitu, J. Laamanen and T. Rüppell

HEP theory group, Dortmund
Helsinki Institute of Physics, Helsinki

The 17th International Conference on Supersymmetry and the Unification of Fundamental Interactions
The muon EDM

\[ \mathcal{L}_{\text{EDM}} = d_l (-i/2) \bar{l}_i \sigma_{\mu \nu} \gamma_5 F^{\mu \nu} l_i \]

- Experimental limit is \( d_\mu = (3.7 \pm 3.4) \cdot 10^{-19} \text{ecm.} \)
- Much higher than \( d_e \rightarrow \) does not constrain flavor violating couplings, which are constrained by rare lepton decay branching ratios.
- Lepton universality puts \( d_\mu \sim 10^{-25} \text{ecm} \) and \( d_\mu^{\text{SM}} \sim 10^{-36} \text{ecm} \).
- Dynamic situation; MEG will push \( \text{Br}(\tau \rightarrow \mu \gamma) \) to \( 10^{-13} \), PSI proposes improvement on \( d_\mu \) by three to six orders of magnitude.
- Assuming the limit \( \text{Br}(\tau \rightarrow \mu \gamma) < 4.5 \cdot 10^{-8} \) how high can \( d_\mu \) be and how much fine tuning is involved.

\[ \mathcal{L} = \bar{l}_i (n_{ijk}^L P_L + n_{ijk}^R P_R) \chi_j^0 \tilde{l}_k + \bar{l}_i (c_{ijk}^L P_L + c_{ijk}^R P_R) \chi_j^- \tilde{\nu}_k + \text{h.c.} \]

\[ T = i e c_{\mu} \frac{q^\nu}{2m_{lj}} \bar{l}_i \sigma_{\mu \nu} (a_{ij}^L P_L + a_{ij}^R P_R) l_j \]
Subamplitudes

\begin{align*}
a_{ij}^L &= \frac{1}{16\pi^2} \sum_{k=1}^{4} \sum_{r=1}^{6} \left( \left( n_{ikr} n_{jkr} \frac{m_{l,j}^2}{m_{\tilde{\chi}_k^0}^2} + n_{ikr} n_{jkr} \frac{m_{l,i}^2}{m_{\tilde{\chi}_k^0}^2} \right) \right. \\
&\quad \times F_1 \left( \frac{m_{l,r}^2}{m_{\tilde{\chi}_k^0}^2} \right) + n_{ikr} n_{jkr} \frac{m_{l,j}^2}{m_{\tilde{\chi}_k^0}^2} F_3 \left( \frac{m_{l,r}^2}{m_{\tilde{\chi}_k^0}^2} \right) \right) \\
&\quad + \frac{1}{16\pi^2} \sum_{k=1}^{2} \sum_{r=1}^{3} \left( \left( c_{ikr} c_{jkr}^* \frac{m_{l,j}^2}{m_{\tilde{\chi}_k^+}^2} + c_{ikr} c_{jkr}^* \frac{m_{l,i}^2}{m_{\tilde{\chi}_k^+}^2} \right) \right. \\
&\quad \times F_2 \left( \frac{m_{\tilde{\nu}_{r}^0}^2}{m_{\tilde{\chi}_k^+}^2} \right) + c_{ikr} c_{jkr}^* \frac{m_{l,j}^2}{m_{\tilde{\chi}_k^+}^2} F_4 \left( \frac{m_{\tilde{\nu}_{r}^0}^2}{m_{\tilde{\chi}_k^+}^2} \right) \right), \\
\end{align*}

\begin{align*}
a_{ij}^R &= a_{ij}^L (L \leftrightarrow R).
\end{align*}
\[ \Delta a_i = \frac{1}{2} \text{Re}(a_{ii}^L + a_{ii}^R), \]
\[ d_i = \frac{e}{4m_{l,i}} \text{Im}(-a_{ii}^L + a_{ii}^R), \]
\[ \Gamma(l_j \rightarrow l_i \gamma) = \frac{\alpha m_{l,j}}{16} (|a_{ij}^L|^2 + |a_{ij}^R|^2). \]

- Non trivial to solve bounds on \( d_i \) from \( \Gamma \).
- Using the mass insertion approximation and parameterizing \( \delta_{LL}^{23} = M_{L23}^2 / m_{\tilde{\mu}}^2, \delta_{RR}^{23} = M_{E23}^2 / m_{\tilde{\mu}}^2 \) the leading diagram is
**Approximate solution**

\[ A_{FLV} = \frac{m_\tau}{m_\mu} \left( -g'^2 (A_{E,22} v_1 - m_\mu \mu \tan \beta) M_1 \right) \times F(M_1^2, m_{\tilde{\mu}_L}^2, m_{\tilde{\mu}_R}^2) \delta_{LL}^{23} \delta_{RR}^{23}, \]

- Here chirality switching contributions (\( \delta_{LR,RL}^{23} \)) are neglected.
- Assuming maximal phases, \( \text{Arg}(\delta_{LL}^{23} \delta_{RR}^{23}) = \pi/2 \), a bound for \( \text{Br}(\tau \to \mu \gamma) \sim a |\delta_{LL}^{23}|^2 + b |\delta_{RR}^{23}|^2 \leq c \) with \( a, b, c > 0 \) can be solved into a maximum of \( d_\mu \).
- For \( M_2 = \mu = 700 \text{ GeV}, M_1 = 350 \text{ GeV}, A = 0 \):

![Graph showing dependence of \( d_\mu \) on \( m_{\tilde{\mu}_L,R} \) for \( \tan \beta = 2.5 \) and \( \tan \beta = 30 \).]
Generic Flavor

- For our numerical analysis, we generate sets of data points by randomly sampling parameters and checking the spectrum, the potential (UFB / CCB) as well as $\Delta a_\mu$.
- Two general scenarios based on properties of the amplitudes

**Light**
- $M_1, M_2, \mu, A_\mu$, diagonal soft masses $M_L$ and $M_E$ and diagonal trilinear soft terms $A_E \leq 1$ TeV.
- Off-diagonal soft masses and A-terms are non-zero only in the $\mu - \tau$ sector and are $\leq 100$ GeV.

**Heavy**
- $M_1, M_2, \mu, A_\mu$, diagonal soft masses $M_L$ and $M_E$ and diagonal trilinear soft terms $A_E \in [3 - 5]$ TeV.
- Off-diagonal soft masses and A-terms are non-zero only in the $\mu - \tau$ sector and are $\leq 3$ TeV.
Specific Flavor

- Hybrid Gauge-Gravity SUSY model (Feng et al. arXiv:0712.0674)
- Gauge mediated contribution are MFV, Gravity mediated contribution are non-MFV parameterized by Gauge-Gravity mixing $x$ and the spurion vev of the horizontal flavor symmetry $\lambda$.

\[
M_L^2 = m_L^2 \mathbf{1} + x m_L^2 X_L, \quad M_E^2 = m_R^2 \mathbf{1} + x m_R^2 X_R.
\]

\[
X_L = \begin{pmatrix}
1 & \lambda^4 & \lambda^8 \\
\lambda^4 & 1 & \lambda^4 e^{-2i\phi_L} \\
\lambda^8 & \lambda^4 e^{2i\phi_L} & 1
\end{pmatrix},
\]

\[
X_R = \begin{pmatrix}
1 & \lambda^2 & \lambda^4 \\
\lambda^2 & 1 & \lambda^2 e^{-2i\phi_R} \\
\lambda^4 & \lambda^2 e^{2i\phi_R} & 1
\end{pmatrix}.
\]
Generic case and Tuning

Logistics

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log(d_{\mu})
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log(\text{Br}(\tau \rightarrow \mu \gamma))
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log(d_{\mu})
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log(\text{Br}(\tau \rightarrow \mu \gamma))
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Hybrid case, Tuning and best point

log\(d_{\mu}\) vs \(\log(\text{Br}(\tau \rightarrow \mu \gamma))\)

log\(d_{\mu}\) vs \(\log(\text{Br}(\tau \rightarrow \mu \gamma))\)

\(\lambda\) vs \(x\)

\(\lambda\) vs \(x\)
Fine Tuning

Classic approach

- Fine tuning is very hard to define absolutely. You recognize it when you see it.
- We compare the random samples and Tuned samples with respect to:
  - Cancellations in the sub amplitudes contributing to the branching ratio.
  - Fine tuning of the phase in the sub amplitudes contributing to the muon EDM.

\[
\begin{align*}
  a_{max} &= \text{largest single contribution to } a_{23}^{L/R} \\
  \text{Br}_{max} &= \text{Br}(\tau \to \mu \gamma) \text{ calculated using only } a_{max} \\
  T_{Br} &= \log(\text{Br}_{max}/\text{Br}(\tau \to \mu \gamma)) \\
  T_{d\mu} &= \log \left( \frac{\text{Im}(a_{22}^R - a_{22}^L)}{\text{Re}(a_{22}^R + a_{22}^L)} \right) .
\end{align*}
\]
Effect of Tuning on $T_{d\mu}$ and $T_{Br}$

Light

Heavy

# of points

$T_{Br}$

$T_{d\mu}$

# of points

$T_{Br}$

$T_{d\mu}$

How fine-tuned is $d_{\mu}$
A different approach

- We consider the relative "volume" of parameter space from which we get acceptable points before and after the Tuning.

![Graph showing the difference in the number of points before and after tuning.

$\phi_{L_{23}} - \phi_{E_{23}}$

- Even though we can still find viable points for all values of $\phi_{L_{23}} - \phi_{E_{23}}$, there are clearly preferred intervals.

- Defining an interval which admits, e.g., 80% of the points as $\bar{X}$ we get a ratio $r = (\bar{X}_{\text{after}} / \bar{X}_{\text{before}})$

- This gives an estimate of how much the preferred range of a parameter has shrunk ($r < 1$) or expanded ($r > 1$).
Correlated Observables and Free Parameters

- Suitable observables must not run up against arbitrary bounds, e.g. the sampling range decided by us.
- Correlations (e.g. phase differences) need special attention.
- For four phases we get six $r$’s from the various differences. The geometric average of the $r$’s gives the reduction in the four dimensional parameter space of the phases as

\[
d_\phi = \prod_i^6 r_i^{4/6}.
\]

- For ratios of diagonal and off-diagonal soft masses we get 36 $r$’s with 13 free parameters

\[
d_M = \prod_i^{36} r_i^{13/36}.
\]

- The overall change in volume is $d_V = r_\beta \cdot d_M \cdot d_\phi$
## Results

<table>
<thead>
<tr>
<th></th>
<th>Light</th>
<th>Heavy</th>
<th>Hybrid</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_\beta$</td>
<td>0.440</td>
<td>1.16</td>
<td>0.392</td>
</tr>
<tr>
<td>$d_\phi$</td>
<td>0.879</td>
<td>0.420</td>
<td>0.455</td>
</tr>
<tr>
<td>$d_M$</td>
<td>$1.69 \cdot 10^{-6}$</td>
<td>$8.46 \cdot 10^{-3}$</td>
<td>$2.86 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>$r_{mL}$</td>
<td>–</td>
<td>–</td>
<td>0.103</td>
</tr>
<tr>
<td>$r_{mR}$</td>
<td>–</td>
<td>–</td>
<td>0.264</td>
</tr>
<tr>
<td>$r_\chi$</td>
<td>–</td>
<td>–</td>
<td>0.448</td>
</tr>
<tr>
<td>$r_\lambda$</td>
<td>–</td>
<td>–</td>
<td>$1.20 \cdot 10^{-2}$*</td>
</tr>
<tr>
<td>$d_V$</td>
<td>$6.52 \cdot 10^{-7}$</td>
<td>$4.12 \cdot 10^{-3}$</td>
<td>$7.46 \cdot 10^{-9}$</td>
</tr>
</tbody>
</table>
Stability of the method

- Our measure of fine tuning depends on how many points we consider to represent a significant part of our sample (e.g. 80%). Defining this fraction as $p$, we can test how stable our method is.
- For the Light (dashed line), Heavy (solid line) and Hybrid model (dotted line / dash-dotted line).
Conclusions

- In a general model $d_\mu \sim 10^{-22}\text{ecm}$ is attainable.
- In specific models of Flavor this may not be possible.
- Fine tuning is required to go above $d_\mu \sim 10^{-24}\text{ecm}$, when taking into account current bounds on $\text{Br}(\tau \rightarrow \mu \gamma)$.
- A heavy SUSY spectrum seems favored from the point of fine tuning.

Thank You!