Antigravitation

Sabine Hossenfelder

Perimeter Institute

Introduction: Objections

- The equivalence principle would be violated.
- If there were negative energies, the vacuum would be unstable.
- A geodesic is independent of the particle’s mass.
- One could construct a perpetuum mobile.
- If there is anti-gravitating matter - where is it?
- And if you have argued it away - why should I care?
Introduction: Objections

- The equivalence principle would be violated.
- If there were negative energies, the vacuum would be unstable.
- A geodesic is independent of the particle’s mass.
- One could construct a perpetuum mobile.
- If there is anti-gravitating matter - where is it?
- And if you have argued it away - why should I care?
Introduction: Objections

- The equivalence principle would be violated.
- If there were negative energies, the vacuum would be unstable.
- A geodesic is independent of the particle’s mass.
- One could construct a perpetuum mobile.
- If there is anti-gravitating matter - where is it?
- And if you have argued it away - why should I care?
Introduction: Objections

- The equivalence principle would be violated.
- If there were negative energies, the vacuum would be unstable.
- A geodesic is independent of the particle’s mass.
- One could construct a perpetuum mobile.
- If there is anti-gravitating matter - where is it?
- And if you have argued it away - why should I care?
- Everybody knows there is no such thing as anti-gravitation.
Motivation: Problems

- Dark Matter
- Dark Energy
- Inflation
- Singularities
- Coincidence problem
- Horizon problem
- Homogeneity problem
- Axis of evil
- ...

Something is missing in our understanding of the universe.
What do I mean with Anti-gravitating matter

- A second copy of the standard model, identical to the one we know, except for its gravitational interaction.
- Both sorts of particles interact only gravitationally.
- In particular, anti-matter has completely normal gravitational properties.
- Disclaimer: This talk is unquantized.
How do we get the funny stuff to move differently?

• Covariant curves are defined via a connection. Yet this connection is unique only after requiring it to be torsion-free and metric-compatible.

• Need second derivative, throw out metric-compatibility.

• Introduce instead second metric $h$ with which the second connection $(h)\nabla$ is compatible.

\[ (g)\nabla, (g)\mathcal{R} \quad \text{with} \quad (g)\nabla g = 0 \]
\[ (h)\nabla, (h)\mathcal{R} \quad \text{with} \quad (h)\nabla h = 0 \]
The Pullovers

- 2nd metric provides another interpretation of the manifold (same manifold, different distance measures) and results in different curves for particles. Its local coordinate basis doesn’t normally coincide with ours.

- Two sorts of indices: $\nu$ raised/lowered with $g$, $\nu$ raised/lowered with $h$

- Pullovers to identify $h$-fields with observables for $g$-observer and vice versa: $P_g, P_h$. Locally linear maps on tensor algebras.

- $h$ is then related to a two-tensor $h = P_h(h)$ and vice versa $g = P_g(g)$.

- Induces pulled-over derivatives by metric-compatibility:

$$P_h^{(h)\nabla A} = (h)\nabla P_h(A)$$

$$P_g^{(g)\nabla A} = (g)\nabla P_g(A)$$
Equations of Motion for Matter Fields

- Action for field

\[ S = \int d^4x \sqrt{-h} \, P_h \left( h^{\nu\kappa} \left( h \nabla_\kappa \phi \right) \nabla_\nu \phi \right) \]

- Leads to

\[ P_h \left( ^{(h)} \nabla^\alpha (h) \nabla_\alpha \phi \right) = 0 \]

Same as

\[ ^{(h)} \nabla^\alpha (h) \nabla_\alpha P_h (\phi) = 0 \]

Same as

\[ ^{(h)} \nabla^\alpha (h) \nabla_\alpha \phi = 0 \]
How to determine the second metric?

- For convenience
  
  \[ g_{\epsilon \lambda} = a_{\epsilon}^{\nu} a_{\lambda}^{\kappa} h_{\nu \kappa}, \quad a_{\epsilon}^{\nu} = [P_g]_{\epsilon}^{\nu} a_{\epsilon}^{\nu} [P_h]_{\nu}^{\nu} \]
  
  \[ g_{\epsilon \lambda} = a_{\epsilon}^{\nu} a_{\lambda}^{\kappa} h_{\nu \kappa}, \quad g_{\epsilon \lambda} = a_{\epsilon}^{\nu} a_{\lambda}^{\nu}, \ h_{\nu \kappa} = a_{\nu}^{\epsilon} a_{\nu \kappa} \]

- The \( a \)'s are not independent: \( \delta a_{\nu \kappa} = \delta a_{\nu \kappa} = 0 \).

- Now use symmetry principle

\[
(g) R_{\nu \kappa} - \frac{1}{2} g_{\nu \kappa} (g) R = T_{\nu \kappa} - |P_h| \sqrt{\frac{h}{g}} a_{\nu}^{\nu} a_{\kappa}^{\nu} T_{\nu \kappa}
\]

\[
(h) R_{\nu \kappa} - \frac{1}{2} h_{\nu \kappa} (h) R = T_{\nu \kappa} - |P_g| \sqrt{\frac{g}{h}} a_{\kappa}^{\nu} a_{\nu}^{\nu} T_{\nu \kappa}
\]

with

\[
T_{\mu \nu} = - \frac{1}{\sqrt{-g}} \delta L \delta g_{\mu \nu} + \frac{1}{2} g_{\mu \nu} L, \quad T_{\nu \kappa} = - \frac{1}{\sqrt{-h}} \delta L \delta h_{\nu \kappa} + \frac{1}{2} h_{\nu \kappa} L
\]

- Degrees of freedom in pull-overs needed to fulfill Bianchi identities.
Action

- Full action

\[ S = \int d^4x \sqrt{-g} \left( \left( g \right) R / 8\pi G + \mathcal{L}(\psi) \right) + \sqrt{-h} P_h(\mathcal{L}(\phi)) \]
\[ + \int d^4x \sqrt{-h} \left( \left( h \right) R / 8\pi G + \mathcal{L}(\phi) \right) + \sqrt{-g} P_g(\mathcal{L}(\psi)) \]

- Dynamical variables \( g \) and \( h \), \( \psi \) and \( \phi \).

- Variation of \( g_{\nu\lambda} h_{\kappa\nu} a^{\nu\kappa} a^{\mu\nu} = \delta^\mu_\lambda \) with \( \delta a^{\nu\kappa} = 0 \) yields

\[ \delta h_{\kappa\lambda} = -[a^{-1}]^\mu_\kappa [a^{-1}]^\nu_\lambda \delta g_{\mu\nu} \]

- Note: There is no negative kinetic energy in the action. Variation over fields (for ‘inertial’ stress-energy) has no change of sign. Sign change only for source term of field equations.

- Both gravitational AND inertial mass (energy) are conserved, thus no vacuum decay possible.
Schwarzschild Metric

• Spherical symmetry with regular particle of mass $M$ in center. Outside solution

\[
\begin{align*}
g_{tt} &= -\left(1 - \frac{2M}{r}\right), \quad g_{rr} = -1/g_{tt}, \quad g_{\theta\theta} = r^2, \quad g_{\phi\phi} = r^2\sin^2\theta \\
h_{tt} &= -\left(1 + \frac{2M}{r}\right), \quad h_{rr} = -1/h_{tt}, \quad h_{\theta\theta} = r^2, \quad h_{\phi\phi} = r^2\sin^2\theta
\end{align*}
\]

and $h_{\kappa\nu} = h_{\kappa\nu}$.

• Does what expected.
Friedmann-Robertson-Walker Metric

- Ansatz for $g$: $ds^2 = -dt^2 + \frac{a^2}{1-ka} (dr^2 + d\Omega^2)$
- Ansatz for $h$: $ds^2 = -dt^2 + \frac{b^2}{1-kb} (dr^2 + d\Omega^2)$
- Yields Friedmann-equations (w/o CC term)

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8}{3} \pi G \left( \rho - |P_h| \left( \frac{b}{a} \right)^3 \rho + \frac{ka}{a^2} \right)$$

$$\frac{\ddot{a}}{a} = \frac{4}{3} \pi G \left( - (\rho + 3p) + |P_h| \left( \frac{b}{a} \right)^3 (\rho + 3p) \right)$$

$$\left( \frac{\dot{b}}{b} \right)^2 = \frac{8}{3} \pi G \left( \rho - |P_g| \left( \frac{a}{b} \right)^3 \rho + \frac{kb}{b^2} \right)$$

$$\frac{\ddot{b}}{b} = \frac{4}{3} \pi G \left( - (\rho + 3p) + |P_g| \left( \frac{a}{b} \right)^3 (\rho + 3p) \right)$$

- Note: anti-graviating matter does not create accelerated expansion.
Relevance?

- Gravitational Lensing?
- Structure of voids?
- Quantize: what about vacuum energy?
Revisited: Objections

- If there were negative energies, the vacuum would be unstable.
  \[ \implies \]  Inertial masses (energies) remain positive.

- A geodesic is independent of the particle’s mass.
  \[ \implies \]  A particle’s curve depends on the connection used.

- One could construct a perpetuum mobile.
  \[ \implies \]  Like charges attract, unlike charges repel: no self-acceleration.

- If there is anti-gravitating matter - where is it?
  \[ \implies \]  As far away as possible.

- And if you have argued it away - why should I care?
  \[ \implies \]  Possibly important at large distances, strong curvature, high densities i.e. astrophysics and cosmology.
Revisited: Objections

- If there were negative energies, the vacuum would be unstable.
  → Inertial masses (energies) remain positive.
- A geodesic is independent of the particle’s mass.
  → A particle’s curve depends on the connection used.
- One could construct a perpetuum mobile.
  → Like charges attract, unlike charges repel: no self-acceleration.
- If there is anti-gravitating matter - where is it?
  → As far away as possible.
- And if you have argued it away - why should I care?
  → Possibly important at large distances, strong curvature, high densities i.e. astrophysics and cosmology.
- Everybody knows there is no such thing as anti-gravitation.