Based on work by Keith Dienes & BT: [arXiv:0806.3364], [arXiv:0811.3335]
So much depends on the vacuum structure of supersymmetric field theories:

How is SUSY broken, if it's broken at all?

What other symmetries are manifest at low energies?

Do metastable vacua exist in addition to the ground state?

If so, what are their lifetimes?

What about vacuum energy?

The point is this: the vacuum structure of any given model plays a significant, and often dominant, role in its phenomenology.
The Uses of Metastable Vacua

- Recently Intriligator, Seiberg, and Shih (ISS) have shown that a large class of models with SUSY-preserving vacua also generically contain metastable vacua where SUSY is broken [hep-th/0602239].

- These vacua frequently have lifetimes longer than the present age of the universe; hence we could be living in a false vacuum.

- This result revealed a vast number of theories that had not been considered phenomenologically viable to be so.
Metastable vacua are also important for other reasons:

Metastable vacua can be populated (even preferentially) in a thermal theory.

They can have a significant impact on studies of the string landscape.

Cosmic strings, domain walls, etc. are often produced when vacuum phase transitions occur and such vacua decay.

You may end up in a metastable vacuum even when you don't want to!

The moral: know thy vacuum structure!
Metastability is everywhere... but subtle.

Metastable vacua appear generically in supersymmetric quantum field theories (as indeed they do in non-supersymmetric ones).

However, while powerful tools like the ADS criterion and the Witten index can be used to analyze the properties of the global minimum of the potential, they don't say much about the presence or properties of local minima.

Consequently, these minima are difficult to locate, enumerate, or analyze in an arbitrary model.
Furthermore, metastable SUSY-breaking models typically involve nonperturbative dynamics in the region separating the metastable minimum from the ground state.

Also, the ground state frequently lies at an infinite distance in field space from the metastable vacuum in the effective, low-energy theory. This must be remedied by some unspecified UV physics.

Reliable vacuum-decay lifetime calculations cannot be performed.

Wouldn't it be nice if there were an alternative scenario...

... that was field-theoretic and incredibly simple?
... in which the dynamics are perturbative everywhere?
... in which lifetimes and transition rates are explicitly calculable?
... for which no ad hoc Planck/String-scale operators are required?
There is: D-Term-Sourced SUSY Breaking!

Here, the ultimate origin of SUSY-breaking is due to the presence of non-trivial Fayet-Iliopoulos terms for additional Abelian gauge groups.

It has all the properties enumerated on the previous slide, plus some added bonuses:

1). Metastable vacua arise **at tree level** and are robust against quantum corrections.

2). It has a **justification from UV physics**, as many of the constructions we discuss here arise generically on the actual string landscape.

**Important:** This is NOT just D-term breaking

In this setup, D-term breaking can source F-term breaking and the breaking of a $U(1)_R$ symmetry.
In this talk, I'll be discussing one particular such scenario:

A simple model with a supersymmetric ground state in which supersymmetry is broken perturbatively, at tree level, in a long-lived metastable vacuum.

Dienes, BT [arXiv:0806.3364]

Other, similar scenarios can give rise to additional features, including (see SUSY '09 talk by K. Dienes)...

… towers of metastable vacua and highly nontrivial systems of vacuum dynamics. Dienes, BT [arXiv:08113335]

… situations in which an arbitrarily large number of exactly degenerate ground states exist. Dienes, BT [to appear]

These are explicit examples of unexplored phenomena in the vacuum structures of SUSY-field theories: engineering complete phenomenological models should be straightforward.
Scenario I: A Moose and a Mass

- Two $U(1)$ gauge groups and five chiral superfields.
- $\Phi_4$ and $\Phi_5$ form a vector-like pair with supersymmetric mass $m$ and are charged under both gauge groups.
- A Fayet-Iliopoulos term for each gauge group: $\xi_1$ and $\xi_2$.
- Gauge coupling constants are taken to be equal: $g_1 = g_2 \equiv g$
- With this R-charge configuration F-term breaking implies R-symmetry breaking.

\[
\begin{array}{|c|ccc|}
\hline
\text{Field} & U(1)_1 & U(1)_2 & R \\
\hline
\Phi_1 & -1 & 0 & 2/3 \\
\Phi_2 & +1 & -1 & 2/3 \\
\Phi_3 & 0 & +1 & 2/3 \\
\Phi_4 & +1 & +1 & 1 \\
\Phi_5 & -1 & -1 & 1 \\
\hline
\end{array}
\]

\[W = \lambda \Phi_1 \Phi_2 \Phi_3 + m \Phi_4 \Phi_5\]
Let us make an explicit choice of parameters to begin with:
\[ \{\lambda, m, \xi_1, \xi_2\} = \{1, 1, 5, 0\} \] (in units of \( m \)).

We will now show that the model defined by this parameter choice contains:

1). A stable, supersymmetric ground state in which R-symmetry is preserved
2). A metastable vacuum in which SUSY and the R-symmetry are both spontaneously broken.
3). A saddle-point barrier between the two.
Stability of the Vacuum

- Extrema are given by

\[ \frac{\partial V}{\partial \phi_i} = \frac{\partial V}{\partial \phi_i^*} = 0 \]

- Stability of the extrema is governed by a $2n \times 2n$ mass matrix.

\[ M^2 = \left( \begin{array}{cc} \frac{\partial^2 V}{\partial \phi_i \partial \phi_j} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j} \\ \frac{\partial^2 V}{\partial \phi_i \partial \phi_j^*} & \frac{\partial^2 V}{\partial \phi_i^* \partial \phi_j^*} \end{array} \right) \]

Scalar Potential

\[ V(\phi) = \frac{1}{2} \sum_{a=1}^{N} g_a^2 D_a^2 + \sum_{i=1}^{n} |F_i|^2 \]

where

\[ D_a = \xi_a + g \sum_{i=1}^{n} Q_{ai} |\phi_i|^2, \quad F_1 = \frac{\partial W}{\partial \phi_i} \]

Diagonalize…

One massless Nambu-Goldstone boson per broken $U(1)$, and a vacuum that’s…

- **Stable** if all other $m^2 > 0$.
- **Unstable** if $\exists m^2 < 0$.

A **flat direction** for each additional $m^2 = 0$. 

Solutions to the Vacuum Equations:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ground State</strong></td>
<td><strong>Metastable Vacuum</strong></td>
<td><strong>Saddle Point</strong></td>
</tr>
<tr>
<td>SUSY: Yes</td>
<td>SUSY: No</td>
<td>SUSY: No</td>
</tr>
<tr>
<td>$R$-Symmetry: Yes</td>
<td>$R$-Symmetry: No</td>
<td>$R$-Symmetry: No</td>
</tr>
<tr>
<td>Gauge Group: $U(1)_2$</td>
<td>Gauge Group: None</td>
<td>Gauge Group: None</td>
</tr>
<tr>
<td>$V_A = 0$</td>
<td>$V_B = 9/2$</td>
<td>$V_C = 45/8$</td>
</tr>
<tr>
<td>$(v_1, v_3, v_5): (\sqrt{5}, 0, 0)$</td>
<td>$(v_1, v_3, v_5): (0, 2, 2)$</td>
<td>$(v_1, v_3, v_5): (\frac{\sqrt{3}}{2}, \frac{\sqrt{7}}{2}, \sqrt{\frac{5}{2}})$</td>
</tr>
</tbody>
</table>
The Picture in Field Space

- Solution C represents the saddle point in field space between solutions A and B through which the classical path runs.
There exists a substantial region of parameter space in which this behavior is realized.

Existence (light) and stability (dark) conditions can be translated into a set of constraints on the model parameters.

The parameter space of the model includes a line of supersymmetric vacua.
The upshot is that our model is not unusual within the parameter space of this model.

When one considers the full landscape of parameter space, similar situations arise between other vacua exist as well.
The Lifetime of the Metastable Vacuum

- In order for our universe to be living in a metastable vacuum, the lifetime of the metastable state must be (at least) on the same order as the age of the universe.

\[
\frac{\Gamma}{[\text{Vol.}]} = Ae^{-B}
\]

\[
B = S_E(\phi) - S_E(\phi_+)
\]

- The decay rate of the false vacuum can be calculated by standard, semiclassical instanton methods [Coleman & De Luccia; Hawking & Moss, et al.].

- Approximate the potential in the region between stable and metastable vacua as triangular [Duncan & Jensen].

- The bounce action can be determined explicitly in this case.
Since all of the relevant regions of vacuum space (including the barrier) involve perturbative physics only, lifetimes for metastable vacua can be calculated reliably.

- The lifetime of the metastable vacuum increases as $\lambda$ increases.
- The $\{35\}$ vacuum is stable on cosmological scales over a large region of parameter space.
Mediation to the Visible Sector

- Mediating supersymmetry breaking to the visible sector in models of this sort is not at all difficult.

- Coupling additional, massive, vector-like pairs of superfields to $U(1)_{1}$ and $U(1)_{2}$ does not disturb the vacuum structure.

- The SUSY-breaking scale is set by $\min(m_{ij})$.

- The additional $\Phi_{i}\Phi_{j}$ pairs can serve as “messenger fields” for gauge mediation, if they have SM gauge charges.

\[ W = \lambda \Phi_{1} \Phi_{2} \Phi_{3} + m_{45} \Phi_{4} \Phi_{5} + m_{67} \Phi_{6} \Phi_{7} + \ldots \]
Summary and Conclusions

- This scenario represents a field-theoretic “kernel” in which supersymmetry and $\mathcal{R}$-symmetry are preserved in the true ground state of the theory, but are broken in a metastable vacuum.

- No nonperturbative dynamics in regions between minima. Transition rates can be reliably calculated, and the lifetime of the metastable vacuum can easily be made to exceed the age of the universe.

- Similar models with more U(1) groups can give rise to more complicated vacuum structures with large or even infinite towers of metastable vacua (cf. SUSY ’09 talk by K. Dienes).